Reduced Order Models for Synchronous Machines

by

Gaber Shabib Salman Ahmad

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

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In

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This thesis is dedicated to my family

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TABLE OF CONTENTS

Page

Page

LIST OF FIGURES

Fig. #		Page

LIST OF TABLES

بسم الله الرحمن الرحيم

موضوع البحث : نماذج مصغرة للآلات المتزامنة

الموجـــــز :

ان الهدف الأساسي لهذه الرسالة هو استنباط نماذج مصغرة متطورة لوحدات التوليد . ان النموذج المصغر يعتبر نموذجا مكافئا للنموذج الأصلي اذا كان الفرق بين نتائج كل من النموذجين تقع في سماحية الخطأ المقبولة في وقـــت معين . ويمكن أن نقول ان النموذج المكافئ يعتمد على :

١ - المعطيات ٢ - النتائج ٣ - فترة التمثيل ٤ - درجة الدقة المطلوبة .

وتعتمد طرق تخفيض درجة النموذج على القيم التحليلية للايجن (الطـــرق الرياضية) للتمثيل الخطي للمعادلات التفاضلية . واحدى هذه الطـــــرق هي الطريقة المستوحاة من كاسترو والأبيض والتي تستخدم دلائل المعطيات والنتائج والتي تمثل قياسات القدرة على السيطرة للمعطيات والملاحظة لنتائج معطـــاه ايضا . ولقد خفضت وحدة التوليد الممثلة بثلاثة عشرة متغير (تمثيل ثنائـــي لعمود الادارة وخماسي التمثيل لملفات المولد، ورباعي التمثيل لمنظم الاشارة الذاتي ، وثنائي التمثيل لأداة ضبط ضغط التربينة) الى احادي وثنائي وثلاثي ورباعي وخماسي التمثيل لكلا من المعطيات والنتائج .

ولقد تم تمثيل النماذج الكلاسيكية الثنائية والثلاثية والرباعيـــــــة والخماسية المبنية على اهمال بعض الظواهر الديناميكية الغير مرغوب فيهـــا وتم مقارنتها بالنماذج الرياضية المقابلة لها . وقد أظهرت المقارنــــة أن التمثيل الرياضي المخفض يعطي دقة حسنة . ولذا فقد أقترحت طريقة لتعديــــل مكونات النماذج الكلاسيكية لكي نحسن من دقتها للوصول بها الى التمثـــــل الرياضي المخفض .

ABSTRACT

The objective of this thesis is the development of reduced order models for generating units. A reduced order model can be considered as an equivalent if the differences between its outputs and the outputs of the complete (high order) model for the same inputs stay within acceptable error tolerance for a given period of time. That is to say the equivalent will depend on (1) Inputs, (2) Outputs, (3) Period of simulation and (4) Degree of acceptable accuracy.

Several methods for model reduction are based on eigenvalue analysis of the linearized system of differential equations. One such method developed by Castro-Leon and El-Abiad utilizes input-output performance indices which are measures of modes controlability to given inputs and modes observablity for given outputs. A thirteenth order model of a generating unit (2 for shaft, 5 for synchronous machine windings, 4 for the AVR-exciter and 2 for Governor-Turbine) is reduced to 1st, 2nd, 3rd, 4th and 5th order for different inputs and outputs.

Also 2nd, 3rd, 4th and 5th order classical models based on neglecting unwanted dynamic phenomena were derived and compared with the corresponding models obtained by mathematical reduction. The comparison showed that the mathematical reduced order models give better accuracies. A method is suggested to modify the parameters of the classical models in order to improve their accuracies to approach that of the mathematical reduced order models.

Chapter 1

INTRODUCTION

1.1 MOTIVATION FOR REDUCED ORDER MODELS

The problem of dynamic stability analysis of a large interconnected power system is extremely time consuming and laborious.

The dynamics of each generating unit are composed of the synchronous machine, automatic voltage regulator, exciter, governor, turbine, energy source system and other control items. When these components are all considered, the order of the overall state space matrix of each unit can reach a relatively large number.

In transient stability studies, the order of each component can be anywhere between the low and high orders given in Table 1.1. Combining all the subsystems, the generating unit can have a relatively high order if all the dynamic phenomena are to be considered. In order to have a small order representation of a generating unit, it is suggested that a modal equivalent of the full representation (high order) be used instead of a partial representation based on ignoring some of the dynamic phenomena. An important point is that when we ignore some dynamic phenomena, the error resulting from the simplified model cannot be calculated without comparing the simplified model

TABLE 1.1. Examples of Low and High Order Modeling for Each Component in a Generating Unit.

Component	Low Order	High Order [1,2]
Shaft	2	10
Generator	0	6
AVR	0	4
Exciter	0	3
P.S.S.	0	3
Governor	0	2
Turbine	0	3
Total	2	31

to the more detailed model. It is necessary to know the error that results from simplification of dynamic models, and to develop a method to minimize this error. The reduced model obtained is called an equivalent or a dynamic equivalent in the case of dynamic models.

1.2 LITERATURE REVIEW

Reducing the order of dynamic systems has recently been investigated by several authors [3,4,5,6,7,8]. The key of their reduction techniques is based on the elimination of system variables which, by inspection, are believed to have negligible effect upon the dynamic response of the system.

The linearized small disturbance equations for a system can be put in the state variables form as

$$\dot{X} = AX + Bu \qquad (1)$$

Applying their methods to a system described by the set of Eqn. (1), the variables associated with relatively small time constants are neglected (i.e. those variables associated with modes whose real part is farthest from the imaginary axis). On the contrary, the variables associated with large time constants are preserved (i.e. those variables associated with modes whose real part is closest to the imaginary axis). The last modes are called the dominant modes.

Davison's [3,4] uses the eigenvalues and eigenvectors of the complete model described by Eqn. (1) to compute a reduced A* matrix of smaller order than the original. Again, this method chooses the dominant eigenvalues as those with the real part closest to the imaginary axis. The reduced matrices are computed as

$$A^{*} = M^{*} \Lambda^{*} M^{*-1} \tag{2}$$

and

$$B^{*} = M^{*} [M_{1}^{-1} B] \tag{3}$$

where M^{*} is a square matrix representing a subset of the complete eigenvector matrix M, its columns are selected based on the dominant eigenvalues. And its rows are selected based on the retained state variables.

Λ^{*} diagonal matrix of the dominant eigenvalues.

M_{1}^{-1} rows corresponding to the retained modes of the inverted eigenvectors matrix.

B Constant matrix of the complete system.

As shown these methods are more applicable to the control area than to power systems.

Al Talib and Krause [5] use the Davison's method to reduce the order of a one machine infinite bus system with excitation control from a tenth to a fourth order system. This is accomplished by first reducing the synchronous machine from seventh to third order and the excitation system from third order to first order and then combining these models to form a reduced order model of the one machine infinite bus system. Finally the reduced system is represented by

$$\dot{X}^{*} = A^{*} X^{*} + B^{*} u \tag{4}$$

where A^{*} and B^{*} reduced constant systems matrices

X^{*} reduced state vector.

In 1971 Kuppurajulu and Elangovan [6] suggested another approach for reducing the order of the A matrix. Two methods were presented.

1- State Variables Grouping Method: This method is based on dividing the system variables into two groups depending on their time constants.

Let the state variables of a system described by Eqn. (1) be partitioned into two groups of state variables. So Eqn. (1) becomes

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \tag{5}$$

where X_1 contains ℓ variables associated with the system's large time constants and X_2 contains $n-\ell$ variables associated with the system's small time constants. The X_1 state variables, because of the large time constants, do not show appreciable variation during the initial stages of transient response and hence they are assumed to retain their initial values during this period.

Accordingly, the model of the system for the initial transient period may be derived from (5) directly by assuming $X_1 = 0$ and $X_1(t) = X_1(0)$, yielding:

$$\dot{X}_2 = a_{22} X_2 + [a_{21} X_1(0) + B_2 u] \tag{6}$$

This is a model of order $(n-\ell)$ and can be used to study the behavior of the highly damped state variables during the initial stages of transient response. During the final stages of the rate of change of the highly damped transients of the variables associated with the small time constants would have disappeared and hence the X_2 variables closely follow the X_1 variables. This justifies the omission of $\dot{X}_2$ terms in the original system of Eqn. (5). The model for the final stages of the transient response may be written as

$$\begin{bmatrix} \dot{X}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \tag{7}$$

which gives

$$\dot{X}^{*} = A^{*}X^{*} + B^{*}u \tag{8}$$

where

$$A^{*} = a_{11} - a_{12}a_{22}^{-1}a_{21} \quad \text{and} \quad B^{*} = B_{1} - a_{12}a_{22}^{-1}B_{2}$$

The order of A^{*} is ℓ and the dominant eigenvalues of the model are determined by solving the characteristic equation, det $(A^{*} - \lambda I) = 0$ where the reduced model eigenvalues closely approximate those of the original system. The advantage of this method is simple because it does not require the exact calculation of eigenvalues and modal matrix of the original system. The main disadvantage of this method is assuming apriori knowledge of the approximate time behavior of the state variables which is very difficult for a large power system.

2. Eigenvalues Grouping Method: This method is an extension of Davison's method, it is based on dividing the original system into a number of subsystems depending upon the location of the eigenvalues in the complex plane.

The advantage of this method is more rigorous than the first method but it needs the calculation of eigenvalues and vectors of the original system.

380

S. Okubo, H. Suzuki and K. Vemura in 1978 [7] presented an applicable method for power systems. This method is based on calculating the significant eigenvalues and eigenvectors in the dynamic stability range of a power system. In this method instead of forming the state space matrix, an operational transfer matrix between internal angles δ and input mechanical torques T_m is used. The dimension of the matrix is equal to the number of generators. The operational transfer matrix is formed as follows.

a. <u>Synchronous Machine</u>

The following equations of synchronous machines are used.

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} -r & X_q(p) \\ -X_d(p) & -r \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ G(p) \end{bmatrix} e_{fd} \tag{9}$$

$$MP^2\delta + DP\delta - GOV(P)\,P\delta = T_m - T_e \tag{10}$$

$$e_{fd} = AVr(p)\,\{-V_t + PSS(p)\,P\delta\} \tag{11}$$

where

AVr(p)	transfer function of an automatic voltage regulator.
PSS(p)	transfer function of a power system stabilizer.
GOV(p)	transfer function of a governor.

b. Load Representation

Nonlinear loads expressed in the following form can be included

$$P_e = \alpha \frac{P_{\ell o}}{V_{\ell to}} V_{\ell t} + \gamma P\theta_\ell \tag{12}$$

$$Q_e = \beta \frac{Q_{\ell o}}{V_{\ell to}} V_{\ell t} + \xi P\theta_\ell \tag{13}$$

It is recognized that α and β represent voltage dependences and that γ and ξ represent frequency dependences of loads.

c. Network Consideration

Network equation of power system is

$$I_{D,Q} = YV_{D,Q} \tag{14}$$

The dimension of Y is n (number of nodes).

Finally, eliminating the variables except δ_k, and T_{mk} (k = 1, ---, m), the transfer matrix between δ_k and T_{mk} is obtained where m is the number of generators. The resultant equation is written in a simplified form as

$$F(p)\,\underline{\delta} = \underline{T}_m \qquad (15)$$

The elements of F(p), that is $f_{ij}(p)$, is function of the operator $p(= \frac{d}{dt})$. The eigenvalues and the eigenvectors of Eqn. (15) can thus be obtained.

In 1978 a good technique applied to power system with a degree of success have been subjected to extensive testing. This work was done by Price [8]. His idea is to divide the power system into a study system and one or more external systems. The study system is that area where disturbances are to be applied and where the response of machines is to be observed. It is represented in the usual manner in the transient stability simulation. The external systems are not of direct interest in the stability study and are important only in so far as they affect the response of the study system to disturbances within the study system. It is therefore desirable to represent the external systems by equivalents.

The buses which connect an external system to the study system or to another external system are referred to as terminals of the external system. The modal analysis dynamic equivalents consist of two stages:

A. Construction of the matrices which represent equivalents of the external systems.

B. Interfacing these matrices with the transient stability simulation of the study system to simulate the complete system.

The equivalent construction procedure actually produces three different forms of equivalents which could be used:

1. Linearised state equations.
2. Diagonalized state equations.
3. Reduced state equations.

The set of differential equation described by Eqn. (1) are linearized about the operating point specified by a load flow of the complete system. The linearized state equations are solved to obtain the eigenvalues and the eigenvectors of the A matrix. Then the diagonalized state equations are obtained by performing a canonical transformation to decouple the modes. Finally the reduced state equations are obtained from the diagonalized state equations by eliminating unwanted modes. The eliminated modes are determined using one or more of the following criteria:

1. Very large eigenvalues.
2. Modes with very small rows in the input matrix (uncontrolled modes).
3. Modes with very small columns in the output matrix (unobserved modes).

1.3 CHARACTERISTICS OF DYNAMIC EQUIVALENTS

Figure 1.1 shows the relation between the original system which has an output y and its equivalent having an output y^*. Normally, a model $(u, y, t_1, t_2, \varepsilon)$ is said to be equivalent if for the same inputs the outputs of the equivalent is the same as the original outputs within an acceptable error. The arguments of the equivalent are defined as follows:

u	input vector (kx1)
y	output vector (mx1)
t_1	initial time of simulation
t_2	final time of simulation
ε	acceptable measure of the error over the period of interest (e.g. root mean square error).

Generally, there are several factors that must be taken into account: the amount of effort spent in computing the reduced model, and whether a more complete (and complex) model is really more convenient than a simpler one, and accuracy considerations: a small increase in order may

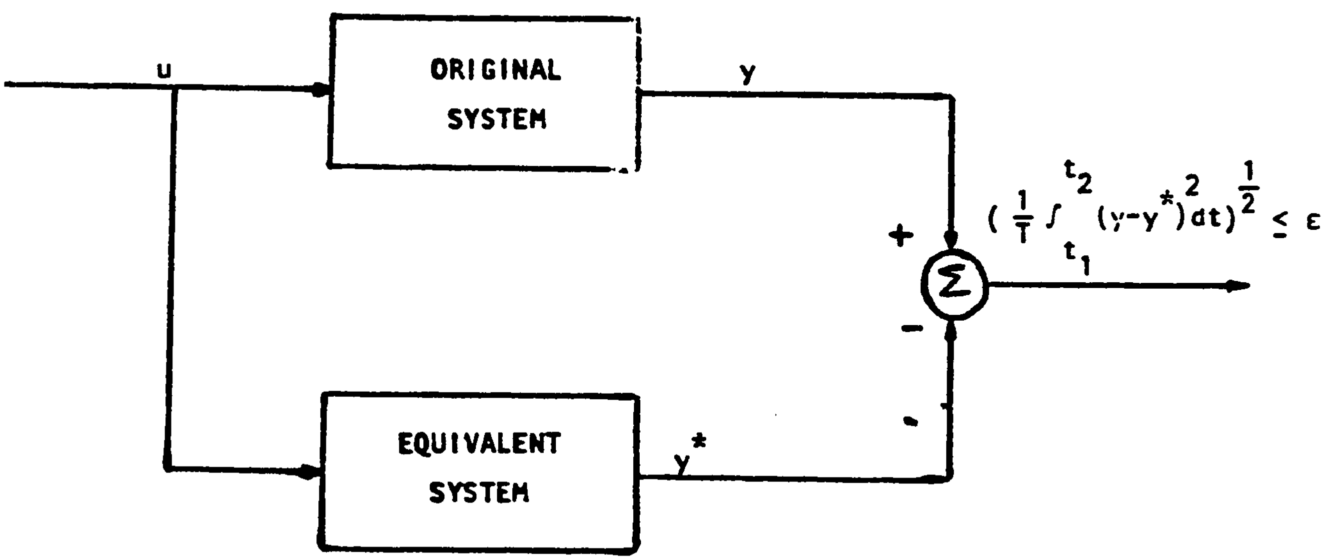


Figure 1.1. The relation between an original system and its equivalent.

bring a far more accurate model and vice-versa. The common denominator is that on making a decision, one must be willing to make tradeoffs. The remarks just mentioned may appear self-evident at this point, but it is not always so when dealing with a specific implementation. To give an example, systems employing the standard state formulation with ABCD matrices, the variables of interest are a few outputs resulting from a small number of inputs (if at all), yet there is usually a large number of states determining the behavior of the system. Most reduction schemes strive to reduce the number of states, and in so doing, it is very easy to forget that we want accurate output variables and not necessarily faithfulness in the retained states, two requirements that are not always dependent. To do so could lead to a higher order equivalent where a lower order would suffice.

An equivalent must be well behaved and should not alter some essential properties of the original system, such as stability: an unstable equivalent of a stable system has no usefulness. At the same time, it should yield consistent results, not good in some cases and poor in others with no apparent reason. Furthermore, it should not be too sensitive to the choice of parameters, especially when the scheme itself does not give clues on which ones are best.

Finally, an equivalencing scheme must be compatible with its intended applications. For example, power system dynamic responses are typically oscillatory and a few schemes do not accommodate this fact very well, a less than careful selection of modes in a modal reduction scheme could yield an equivalent having complex coefficients and complex numbers in the results where a purely real response is sought. Moreover, most integration routines are not amenable in this regard, and those which are amenable are slow at best.

1.4 APPROACH AND THESIS SURVEY

In this thesis the Generating Unit is represented in detail. Small displacement technique is employed to linearize system equations about an operating point. Although this approach is widely used in power system stability studies, there is no guarantee that the results obtained are actually applicable to the original nonlinear system. The reduced order models are designed using Davison's reduction method with the modification that the retained modes of the dynamic model are identified using the method of input-output performance indices. The method is simple and practical, it depends on calculating the input-output performance index for each mode, then the modes with highest index are selected as important instead of selecting the modes closest to the imaginary axis.

The classical methods for simplification are used to design a simplified model based on engineering experience in this field. Finally, engineering judgement is used to modify the simplified model parameters in order to force its response to approximate that of the modal reduction equivalent.

The present chapter gives an overview of modal reduction methods, their advantages and their disadvantages and the characteristic of an equivalent. Chapter two derives the method used to calculate the input-output index, analysis of reduced model, error analysis of the reduced models, calculation of the output of the detailed model and the reduced models.

Chapter three presents an analysis of a one machine connected to infinite bus system through a transmission line to test the method explained in Chapter 2 taking into consideration the different inputs of the system. Chapter four uses the engineering experience to compare the numerical results obtained from Chapter three for different reduced models to the classical Engineering models and how to improve the response of these models to satisfy the desirable constraints.

Chapter 2

MODAL DYNAMIC EQUIVALENT

2.1 INTRODUCTION

The development of EHV systems during the 1960's greatly expanded the size of system representations required for steady state and dynamic simulation studies. With today's heavily interconnected systems, and the necessity for more extensive and sophisticated planning studies, utilities are being forced to use larger load flow and transient stability computer programs requiring larger blocks of core and increasing computation times. Such expansion in the scope of load flow and transient stability investigation have happened without extensive use of equivalents. Nevertheless, for transient stability studies, the necessity of representing large numbers of synchronous machines itself has become a problem.

Reduction methods by developing equivalents of active components in the system representation are often used. A disadvantage is the difficulty in assessing the error incurred; also, some fundamental properties could be inadvertently hidden by neglecting some components. Investigation of dynamic equivalents is of concern in several disciplines.

Most methods previously developed are concerned mainly with a reduced equivalent that retains the same dominant eigenvalues as the complete model, where dominant eigenvalues are defined as those whose real parts are closest to the imaginary axis (i.e. distance of the eigenvalues from the imaginary axis). In this chapter, the concept is generalized into input–output performance index [9]. The dominant modes are selected based on a mathematical instead of intuitive bases. It is shown that dynamic equivalents, and the corresponding bounds in the response relative to the full system, can be determined in terms of the input–output performance index (T–factors). No restrictions on the structure of the state equations or on the applicability of the procedures are imposed.

2.2 SOLUTION OF THE STATE EQUATIONS

The dynamic system is represented by a set of differential equations in the form:

$$\dot{X} = AX + Bu \tag{2.1}$$

$$y = CX + Du \tag{2.2}$$

where

A, B, C and D are constant system matrices depending on the parameters of the system and have the dimensions of nxn, nxk, mxn and mxk respectively.

X state vector nx1

u input vector kx1

y output vector mx1

The behavior of some or all states can be treated as particular cases when the rows of C are made of rows of the unit matrix. The solution of 2.1 in time domain is

$$X(t) = e^{At} X(o) + \int_o^t e^{A(t-\tau)} Bu(\tau)\, d\tau \tag{2.3}$$

where

The zero input response is:

$$e^{At} X(o) \tag{2.4}$$

and the zero state response is:

$$\int_o^t e^{A(t-\tau)} Bu(\tau)\, d\tau \tag{2.5}$$

The set of differential equations described by (2.1) are the result of linearization about an operating point of originally a nonlinear system.

The states are then all small displacement states with zero displacement at time zero. Thus, letting the initial state vector equal to zero. Equation (2.3) reduces to;

$$X(t) = {}_0\int^t e^{A(t-\tau)}\, Bu(\tau)\, d\tau \qquad (2.6)$$

Another assumption can be added to Eqn. (2.6) yields a simple set of equations which assumes that the inputs to the system are step functions. The reason for that is to investigate the overshoots and the settling times. So, Eqn. (2.6) becomes

$$X(t) = {}_0\int^t e^{A(t-\tau)}\, BU\, d\tau \qquad (2.7)$$

or

$$X(t) = e^{At}\, {}_0\int^t e^{-A\tau}\, d\tau\, BU \qquad (2.8)$$

integrating:

$$X(t) = -A^{-1}\,(I - e^{At})\, BU \qquad (2.9)$$

use

$$A = M\, \Lambda\, M^{-1}$$

$$A^{-1} = M\Lambda^{-1}M^{-1} \qquad (2.10)$$

and $\qquad e^{At} = M\, e^{\Lambda t}M^{-1}$

where

M matrix of eigenvectors

Λ diagonal matrix of eigenvalues assuming unrepeated eigenvalues.

Substituting (2.10) in (2.9) and rearranging Eqn. (2.9) yields to,

$$X(t) = -M\Lambda^{-1}(I - e^{\Lambda t})\, M^{-1}\, BU \tag{2.11}$$

The output vector is

$$y(t) = -CM\Lambda^{-1}(I - e^{\Lambda t})\, M^{-1}BU + DU \tag{2.12}$$

2.3 INPUT-OUTPUT PERFORMANCE INDEX [T FACTORS]

Set D = 0 in Eqn. (2.12) and identifying the contribution of a mode to a given output. Then the output y_i is equal to

$$y_i(t) = \sum_{\ell=1}^{k} [-(\sum_{k=1}^{n} C_{i,k} \cdot M_{k,j} \cdot \sum_{L=1}^{n} N_{j,L} \cdot B_{L,\ell}) \cdot \frac{(1-e^{\lambda_j t})}{\lambda_j}]\, U_\ell \tag{2.13}$$

where

$$N = M^{-1}$$

Let us give an attention to the multiplication of the matrices C*M and N*B. Our objective is to relate this to the control area. Since we know that the observability of a system is investigated from the output matrix, while the controllability of a system is investigated from the input matrix. So, let m_j be a column in the eigenvector matrix M, and m_p be another column in the eigenvector matrix M. If $C_i\, m_j \ll C_i\, m_p$ $j \neq p$, this means that mode j is weakly observable in y_i (Note that this definition applies to any set of conformable vectors and modes as well). Similarly, define n_j is a row in the matrix N, and n_p is another row in the matrix N, if $n_j B_k \ll n_p B_k$; $j \neq p$, then mode j is said to be weakly controllable by u_k. Thus the relative magnitudes of $C_i m_j$ and $n_j B_k$ give an indication to the effective mode among all modes.

The idea is extended to define input-output performance index composed from a controllability and observability point of view:

$$T_{ijk} = -\frac{C_i\, m_j\, n_j\, B_k}{\lambda_j} \tag{2.14}$$

where $C_i m_j$ observability factor

$n_j B_k$ controllability factor

or in matrix form

$$T_{ijk} = \quad -\left(\sum_{k=1}^{n} C_{i,k} M_{k,j} \cdot \sum_{\ell=1}^{n} N_{j,\ell}\, B_{\ell,k}\right)/\lambda_j \tag{2.15}$$

$i = 1,m$

$j = 1,\dots n$

$k = 1, \dots k$

$T_{i,j,k}$ is the input-output performance index, and represents the contribution of mode j to output i from input k. The T-factors give an indication of how input U_k affects the output y_i through mode j. Normalization by λ_j gives more weight to the eigenvalues close to the origin. The output y_i is then:

$$y_i(t) = \sum_{\ell=1}^{k} \sum_{j=1}^{n} T_{i,j,\ell} (1 - e^{\lambda_j t}) U_\ell \qquad i = 1,m \qquad (2.16)$$

2.4 INPUT-OUTPUT PERFORMANCE INDEX FOR DIFFERENT MODE

If the A matrix is real unsymmetric then the eigenvalues can be real or real and complex conjugate pairs defined as $\sigma \pm j\omega$, where σ is the damping factor and ω is the phase.

From Eqn. (2.15) the T-factor corresponding to a real mode is real and those corresponding to a complex conjugate pair of eigenvalues is a complex conjugate pair. The output takes different cases according to this phenomena.

2.4.1 Oscillatory Mode Pairs

The output of an oscillatory pair is defined as follows:

$$y_i(t) = [(R_e(T) + JIm(T))(1-e^{(\sigma+j\omega)t}) + (R_e(T) - JIm(T))(1-e^{(\sigma-j\omega)t})]\, U \qquad (2.17)$$

36b

after multiplication we get;

$$y_i(t) = [2R_e(T) - 2e^{\sigma t}(R_e(T)\ \mathrm{Cos}\ \omega t - \mathrm{Im}(T)\sin \omega t)]\ U \qquad (2.18)$$

2.4.2 Real Eigenvalue

The output for this eigenvalue is

$$y_i(t) = R_e(T)\ (1 - e^{\sigma t})\ U \qquad (2.19)$$

at steady state ($t \to \infty$)

$$y_i(\infty) = R_e(T)\ U \qquad (2.20)$$

2.5 MODE DOMINANCE INDICES

Equation (2.16) shows that an output y_i involves the summation of all input-output performance indices. So, any mode deleted from the output equation it considers as an error. To yield an accurate modal equivalent it is necessary to incorporate all modes with highest input-output indices. According to this, the important modes are the modes having large input-output indices. The magnitude of the input-output index can be calculated as:

$$|T_{i,j,k}| = (|\mathrm{Re}(T_{i,j,k})|^2 + |\mathrm{Im}(T_{i,j,k})|^2)^{1/2} \qquad (2.21)$$

Now only a minimum subset of modes, necessary to reproduce the behavior of the selected output within error constraints is preserved. In case of multi-output system, it is difficult to satisfy all constraints for all the outputs of the system by one small modal equivalent. To give an example, in an electric power system simulation, a dynamic equivalent designed to yield accurate generator rotor speeds and angles will be poor in reproducing rotor currents. There are two options: one is to increase the size of the equivalent, to include additional modes so more accurate currents can be obtained, the other option is to develop another equivalent designed to yield good currents. In this case, we would have two realizations of the same system, designed to meet two different criteria.

2.6 MODAL REDUCTION TECHNIQUE

Several authors [3,4,5,6,7,8] have discussed methods of reducing dynamic systems in which certain eigenvalues and elements of eigenvectors of the original system are retained. The differences between them are the way to choose the dominant eigenvalues. According to this thesis the most suited method to incorporate the input-output performance indices is Davison's method [3,4]. Instead of choosing the eigenvalues closest to the $j\omega$-axis, the eigenvalues which have highest input-output indices can be selected and the same procedure can be continued to reduce a system of higher order to one smaller order while retaining the main dynamic response of the original system.

2.6.1 Davison's Method

The system described by Eqn. (2.1) can be reduced to another one which has a smaller order described by the set of differential equation as

$$\dot{X}^{*} = A^{*} X^{*} + B^{*} U \qquad (2.22)$$

where

X^{*t} $= [X_r \, X_s \, X_p \, \text{----} \, X_\ell]$

A^{*} $= \ell x \ell$ matrix of the reduced system

B^{*} $= \ell x k$ matrix of the reduced system

and $\quad r, s, p, \text{----}, \ell$ retained state variables

$$A^{*} = M^{*} \Lambda^{*} M^{*-1} \qquad (2.23)$$

$$B^{*} = M^{*} [M^{-1}B]^{*} \qquad (2.24)$$

where

$[M^{-1}B]^{*}$ is an $\ell x K$ matrix consisting of the retained ℓ rows of $M^{-1}B$ corresponding to X^{*}.

The previous matrices are formed as Davison's method in [3,4] with the modification in the construction of the matrix M^{*}.

2.6.2 Construction of the Reduced Matrix M^*

The matrix M^* is constructed by selecting some rows and columns from the eigenvector matrix M of the complete system:

a) The columns of the matrix M^* are selected based on the retained eigenvalues obtained from the input-output performance indices.

b) The rows of the matrix M^* are selected based on the interesting state variables in the system depending on retained eigenvalues. Davison's method suggested another point here that the variables which be selected should yield a nonsingular matrix M^* (i.e. Det $M^* \neq 0$).

This last point is difficult, it implies a trial and error method should be accomplished. It is solved in this thesis by using engineering experience choosing variables that are important or using sensitivity analysis to relate eigenvalues to state variables [10,11]. Equation (2.22) is solved as (2.1) was, and the solution gives the response of the states of the reduced system.

2.7 OUTPUT OF REDUCED ORDER SYSTEM

By solving Eqn. (2.22) the output of the reduced system can be calculated

$$X^{*}(t) = {}_{o}\int^{t} e^{A^{*}(t-\tau)}\, B^{*} U \, d\tau \tag{2.25}$$

$$X^{*}(t) = -M^{*} \Lambda^{*-1} (I - e^{\Lambda^{*} t})\, M^{*-1}\, B^{*} U \tag{2.26}$$

Substituting (2.24) for B^{*} yields

$$X^{*}(t) = -M^{*} \Lambda^{*-1}\, (I - e^{\Lambda^{*} t})\, [M^{-1} B]^{*}\, U \tag{2.27}$$

Now, an input-output performance index can be calculated using Eqn. (2.27) in a method identical to that performed earlier for the complete system. The results show that the input-output indices for the reduced model are identical to the input-output indices of the retained modes of the complete model. The output of the reduced model is subset from the output of the complete system as,

$$y_i^{*}(t) = \sum_{j \epsilon z} T_{i,j,k}\, (1 - e^{\lambda_j t})\, U_k \tag{2.28}$$

where z indicates a subset of the retained modes.

Noting that the input functions can be step function or other suitable functions.

2.8 ERROR ANALYSIS

The error of the reduced model compared to the complete model can be calculated using Eqns. (2.16) and (2.28).

$$E_i = y_i - y_i^* \tag{2.29}$$

$$|E_i| = \frac{1}{T} \sum_{t_1}^{t_2} | y_i(t) - y_i^*(t) | \tag{2.30}$$

where

t_1 starting time

t_2 ending time

T study time $t_2 - t_1$

Equation (2.30) can be modified in another form to calculate the R.M.S. error.

$$\text{R.M.S. Error of } y_i = \left[\frac{1}{T} \sum_{t_1}^{t_2} (y_i(t) - y_i^*(t))^2 \right]^{1/2} \tag{2.31}$$

The error resultant from Eqn. (2.29) can be plotted showing the error of the reduced model at any time and is a meaningful if it is drawn below the complete response.

The resultant error sometimes is used to find a criterion for the accuracy of each reduced order model. To handle this case the error obtained above should be normalized to certain basis. So, a more meaningful expression for the error is the r.m.s. error normalized by the r.m.s. output of the complete system. The p.u. R.M.S. is calculated as

$$\text{p.u. R.M.S. Error of } y_i = \left[\frac{\sum_{t_1}^{t_2} (y_i(t) - y_i^*(t))^2}{\sum_{t_1}^{t_2} (y_i(t))^2} \right]^{1/2} \qquad (2.32)$$

where T is the period of time over which the error is calculated and equal to $(t_2 - t_1)$, which is the period of interest for a dynamic study for which the reduced model is intended.

Chapter 3

REDUCTION MODEL OF A MACHINE CONNECTED TO AN INFINITE BUS SYSTEM

3.1 INTRODUCTION

The algorithm discussed in chapter 2 is tested for a machine connected to an infinite bus bar through a transmission line. Figure 3.1 shows a diagram of this model. Appendix A3-I calculate the operating point conditions for the synchronous machine.

The system main components consist of a 5th order winding representation for the synchronous machine, a 4th order automatic voltage regulator (AVR) and exciter, a second order shaft and a second order turbine and governor, yielding a total of 13 differential equations. In Table 3.1 the parameters of the complete unit are listed along with the operating point conditions. A block diagram showing dynamic subassemblies is in Fig. 3.2. The first step in the analysis is to form the A and B matrices of the uncoupled system, including the nondynamic variables which are used by the transformation matrices. Appendix (A3-II) shows the detailed model of generator windings, AVR and exciter, shaft and turbine system.

3.2 COMPLETE GENERATING UNIT MODEL

Appenidx (A3-III) shows a complete generating unit model connected to infinite bus bar. Also in this model the terminal voltage V_t is expressed in

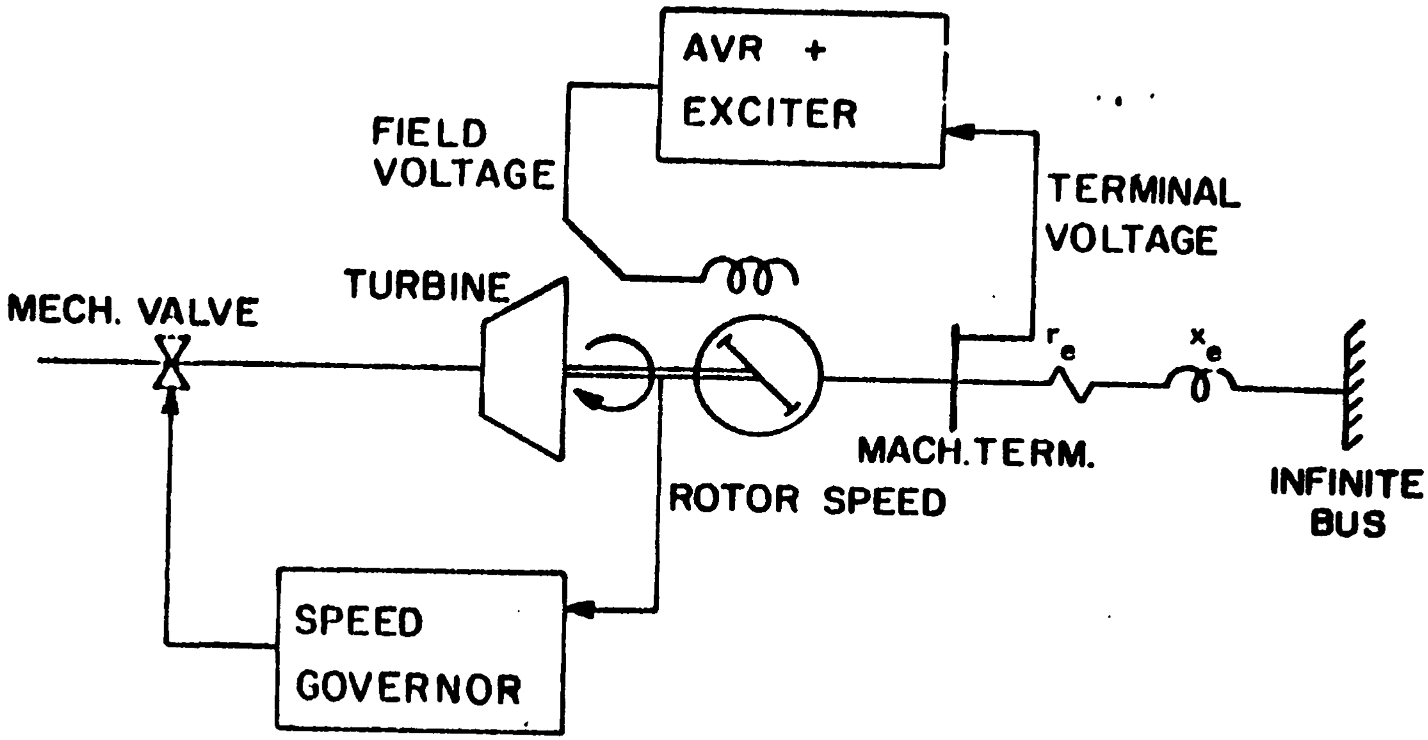


Figure 3.1. Power System Basic Unit Represented by a Single Machine Infinite Bus-Bar Model

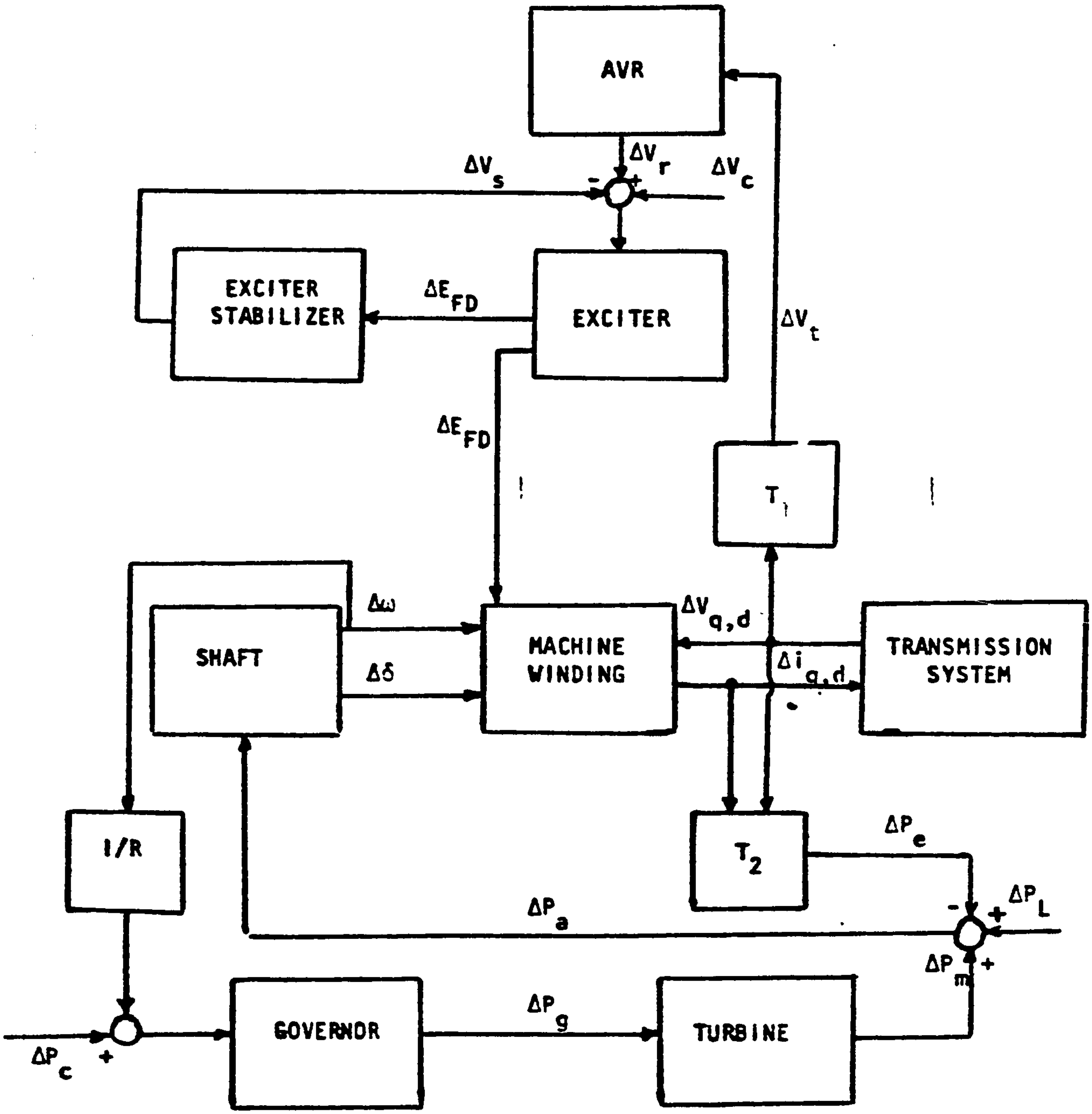


Figure 3.2. One machine infinite bus system dynamic subassemblies.

360

terms of state variables. The choice of the state variables is arbitrary and are considered to be

$$X^t = [\Delta i_q, \Delta i_d, \Delta i_{kq}, \Delta i_{kd}, \Delta i_f, \Delta\delta, \Delta\omega, \Delta E_{FD}, \Delta V_a, \Delta V_r, \Delta V_s, \Delta P_m, \Delta P_g]$$

The parameters of the A matrix change as the operating point changes. This implies that the matrix A takes different values as the loading conditions change. Generally the following points affect the A matrix:

1- Network parameters.

2- quiescent operating conditions.

3- infinite bus bar voltage.

The numerical value of the coupled A matrix is listed in Fig. 3.3. The input into the system is considered to be the change in load power, the setting of the exciter reference voltage and the setting of the governor power.

The interesting outputs are considered to be the rotor angle, the rotor frequency, the output of the exciter voltage and the stator current components. These outputs are selected based on the engineering experience in this field. Normally the important ones are the shaft dynamics which are the rotor angle and the rotor frequency followed by the effect of the exciter in the direct axis of the synchronous machine and finally the effect of the stator dynamics. According to these, each row in the output matrix C is equal zero except that the element corresponding to the variable considered as output is equal unity.

TABLE 3.1. Parameters of One Machine-Infinite Bus Model (in Per Unit).

MACHINE DATA

$r_a = .0032$

$r_{kq} = .014$

$r_{kd} = .011$

$r_{fd} = .001$

$X_{mq} = 1.47$

$X_{md} = 1.56$

$X_{\ell a} = .093$

$X_{\ell kq} = .032$

$X_{\ell kd} = .048$

$X_{\ell fd} = .086$

TRANSMISSION LINE DATA

$r_e = 0.02$

$X_e = .2$

TURBINE AND SHAFT DATA

$M = .014\ (\text{sec})^2$

$D = 0$

$T_g = .25$ (sec)

$T_T = 1.0$ (sec)

$R = .05 \times 120\pi$

OPERATING POINT

$V_{inf.bus} = 1.0 + j0$

$V_t = 1.36 + j.148$

$V_{qo} = .825$

$V_{do} = .564$

$V_{tqo} = 1.03$

$V_{tdo} = 0.51$

$I_{qo} = 0.332$

$I_{do} = .943$

$\delta_o = 34.3^o$

$V_{fo} = 2.585$

$I_{fo} = 1.56$

AVR AND EXCITER DATA

$T_e = 2.028$ (sec)

$T_a = 0.02$ (sec)

$T_r = .001$ (sec)

$T_s = .45$ (sec)

$K_e = 13.89$

$K_a = 50$

$K_r = 1$

$K_s = .057$

-26.9686	-2153.9976	-15.9274	1813.4043	1813.4043	655.0181	2.0988	0.000	0.000	0.0000	0.0000	0.000	0.000
2056.4148	-27.0612	-1714.6516	-8.0752	-0.4097	-963.6167	-1.7564	0.263	0.000	0.0000	0.0000	0.000	0.000
-26.3940	-2108.1067	-19.1021	1774.7695	1774.7695	641.0627	2.0540	0.000	0.000	0.0000	0.0000	0.000	0.000
1294.2375	-17.0314	-1079.1433	-37.1039	2.5011	-606.4675	-1.1054	-1.603	0.000	0.0000	0.0000	0.000	0.000
722.3555	-9.5058	-602.3044	27.5120	-2.9878	-338.4890	-0.6170	1.915	0.000	0.0000	0.0000	0.000	0.000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.000	0.000	0.0000	0.0000	0.000	0.000
-167.7471	2.0700	99.3299	-35.8800	-35.8800	0.0000	0.0000	0.000	0.000	0.0000	0.0000	71.429	0.000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.493	6.849	0.0000	0.0000	0.000	0.000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.000	-50.000	-2499.9995	-2499.9995	0.000	0.000
406.9326	-855.9734	-417.8723	874.1760	876.0095	-54.7003	0.9738	0.063	0.000	-999.9995	0.0000	0.000	0.000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.063	0.868	0.0000	-2.2222	0.000	0.000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.000	0.000	0.0000	0.0000	-1.000	1.000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.2122	0.000	0.000	0.0000	0.0000	0.000	-4.000

Figure 3.3. A matrix of 13th order system

3.3 EIGENVALUES ANALYSIS

Table 3.2 shows the eigenvalues of matrix A. Since the matrix A is real unsymmetric, its eigenvalues are either real or complex conjugate pairs. Note also that the real parts of all the eigenvalues are negative, which means that the system is stable under the conditions assumed in the developement of this model (i.e. small perturbation about a quiescent operating condition). The imaginary parts of complex conjugate pairs of eigenvalues, with dimensions of rad/sec, indicate a frequency of oscillation which is damped if the real parts, called the damping coefficients, are negative. Moreover, the value of the damping coefficient is a measure of system damping. The reciprocal of the absolute value of damping coefficient gives the corresponding time constant, hence, it is a measure of the time required for the system to reach steady state condition. Thus, the smaller the magnitude of real part of an eigenvalue the longer the corresponding sinusoidal time function may persist, and this may constitute an undesirable response.

The complex conjugate pairs of modes 2,3 and 6,7 are poorly damped. This aspect is important because most cases of poor damping problems in a power system appear in low frequency modes. This is clear because these pairs have frequencies approximately a fraction of Hz and a few Hz respectively. The other complex pairs correspond to very fast transient response since these pairs have large real parts. One of these pairs has a frequency of about 60 Hz; this is the 60 Hz component injected into the rotor

TABLE 3.2. The eigenvalue of the A matrix described in Table.

Mode	Eigenvalues
1	-.9680
2,3	-.6610+j1.01
4	-4.137
5	-15.14
6,7	-2.574±j12.854
8	-38.25
9,10	-26.24±j39.7
11,13	-26.75±j376.0
12	-999.99

circuits to balance the MMF caused by stator DC currents. Generally, this analysis lead us to solve for Eigenvalues Sensitivity analysis (i.e. partial derivatives of eigenvalues with respect to elements of the Matrix A as well as with respect to system parameters).

3.4 CASE 1 – THE INPUT IS CONSIDERED TO BE THE CHANGE IN THE LOAD POWER

Table 3.3 shows the input-output performance indices for all modes for each output. The outputs are considered to be the rotor angle, the rotor frequency, the output voltage of the exciter and the stator current components. The table shows also the ranking of all modes for each output. We stopped ranking the modes for each output if the input-output performance index are very small or zero. This means that these modes are less important and ignoring it does not affect the result.

The complete model for the generating unit has been reduced to models of smaller order using the technique given in chapter two.

The modes which are found to be dominant from the thirteenth order system are the oscillating pair 6, 7 which is associated with a hunting frequency of approximately 2.0 Hz, the oscillating pair 2,3 which have a lower frequencies of about less than one Hz which are related to the interaction between the exciter and the direct axis of the synchronous machine, and finally the real pole due to mode 5. The previous modes are the most important for all the five outputs but the ranking order is not the same for each of them. From Table 3.3 the smallest reduced model that can be obtained for the rotor angle is a second order from modes 6 and 7, or a fourth order from modes 6,7 and 2,3 since modes 2,3 are complex conjugate pair of eigenvalues. This does not imply that a third order model from modes 6,7 and 5 is not accurate but we expect from the input-output performance indices that the error would not be much reduced than in the case of a second order model. Still, a fifth order from modes 2,3, 6,7 and 5 yields a result that is practically equivalent to the complete system. The direct axis current is similar to the rotor angle. The difference is for the rotor frequency, quadrature axis current and the field voltage, for a second order from modes 6,7 are accepted with more error but third order from modes 6,7 and mode 5 are accepted since mode 5 comes before the oscillatory complex conjugate pair 2,3 so a fourth order reduced model is not accurate. Finally a fifth order from modes 2,3, 6,7 and 5 yields results that are practically equivalent to the thirteen order system. For field voltage modes 2,3 come before mode 6,7 so a third order from modes 2,3 and 5 is better than a third order from 6,7 and 5.

For each model the input-output index indicated the important modes. It should be noted that the modes that were selected were not all the ones closest to the imaginary axis. Therefore the value of the input-output index has been shown.

TABLE 3.3. Input - Output performance indices to a change in the load power.

Mode	Elegenvalue	Rank	Delta	Rank	Frequency	Rank	Field voltage	Rank	IQ	Rank	ID
1	-.9680 +j 0.000		.0150 + j 0.000		-.0150 +j 0.000		0.0360 +j 0.000		0.0060 +j 0.000		0.032 +j 0.000
2	-.6610 +j 1.018	3,4	.0840 - j 0.097	4,5	-.1550 -j 0.021	1,2	0.5650 +j 0.414	4,5	0.0210 -j 0.047	3,4	-0.113 +j 0.164
3	-.6610 -j 1.018		.0840 + j 0.097		-.1550 +j 0.021		0.5650 -j 0.414		0.0210 +j 0.047		-0.113 -j 0.164
4	-4.137 +j 0.000		-.0160 + j 0.000		.0670 +j 0.000		0.0170 +j 0.000		0.0020 +j 0.000		-0.035 +j 0.000
5	-15.14 +j 0.000	5	.0560 + j 0.000	3	-.8440 +j 0.000	3	-.4320 +j 0.000	1	-.4190 +j 0.000	5	0.135 +j 0.000
6	-2.574 +j 12.85	1,2	.2290 - j 0.089	1,2	.5550 +j 3.177	4,5	0.0010 +j 0.186	2,3	0.3490 +j 0.196	1,2	0.362 -j 0.122
7	-2.574 -j 12.85		.2290 + j 0.089		.5550 -j 3.177		0.0010 -j 0.186		0.3490 -j 0.196		0.362 +j 0.122
8	-38.25 +j 0.000		.0000 + j 0.000		-.0090 +j 0.000		0.0140 +j 0.000		0.0000 +j 0.000		0.008 +j 0.000
9	-26.24 +j 39.78		.0000 + j 0.000		0.0000 +j 0.000		0.0020 +j 0.014		0.0000 +j 0.000		0.000 +j 0.000
10	-26.24 -j 39.78		.0000 + j 0.000		0.0000 +j 0.000		0.0020 -j 0.014		0.0000 +j 0.000		0.000 +j 0.000
11	-26.75 +j376.03		.0000 + j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.0000 -j 0.001		-.001 +j 0.000
12	-26.75 -j376.03		.0000 + j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.0000 +j 0.001		-.001 +j 0.000
13	-999.99-j 0.000		.0000 + j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.000 +j 0.000

It should be noted that for each mode an associated variable is chosen. The way to do that is to solve for the sensitivity. However, engineering experience has revealed which variables are the most important ones among the thirteen variables.

Tables 3.4 and 3.5 show the R.M.S. error and the p.u. R.M.S. error of the reduced models which are calculated over a four second period using the error formula derived in chapter 2 respectively. The p.u. R.M.S. error can be used as a criterion in ranking the accurate reduced order models for each output. The reduced order models are differentiated by a digit showing the order of the reduced model and a character showing the reduced model.

The output of the reduced order models are calculated and plotted versus the complete response of 13th order model to a 1.0 %. step change in the load power, also the error of the reduced order models are plotted below each output over the same studied period in Figs. 3.4 through 3.13.

3.5 CASE 2: THE INPUT IS CONSIDERED TO BE THE CHANGE IN THE SETTING OF THE GOVERNOR POWER

Table 3.6 shows the input-output performance indices for this case. As a repeat for the previous case, the modes which are found to be important in the complete system are modes 1, 4, 5, the oscillating pairs of modes 2, 3 and modes 6, 7. These oscillating pairs have a lower frequency of

TABLE 3.4. R.M.S % Error of reduced models of 13th order system for 1. % change in the load power.

ORDER OF MODEL	DELTA	FREQUENCY	FIELD VOLTAGE	ID	IQ	PRESERVED EIGENVALUES
2d	----	1.065737	----	----	0.352741	6,7
2e	0.244566	1.065737	----	----	----	6,7
2f	----	----	1.300532	0.57968	----	6,7
3a	0.190733	0.246564	----	----	0.063332	6,7,5
3b	0.190733	0.246564	----	0.29191	----	6,7,5
3bb	0.059862	0.901432	----	0.109676	----	reduced from 7th order.
3c	----	1.579461	0.088112	0.708764	----	2,3,5
4a	0.051668	0.790031	----	0.115354	0.408354	6,7,2,3
4b	0.051668	0.790031	0.365638	0.115354	----	6,7,2,3
5a	0.004207	0.044373	0.062151	0.019213	0.005726	6,7,2,3,5

Table 3.5. P.U. R.M.S % Error of reduced models of 13th order system for 1. % change in the load power.

ORDER OF MODEL	DELTA	FREQUENCY	FIELD VOLTAGE	ID	IQ	PRESERVED EIGENVALUES
2d	----	1.040226	----	----	0.964565	6,7
2e	0.349054	1.040234	----	----	----	6,7
2f	----	----	1.943425	0.281393	----	6,7
3a	0.272234	0.240635	----	----	0.173244	6,7,5
3b	0.272234	0.240635	----	0.500901	----	6,7,5
3bb	0.077004	0.879133	----	0.221117	----	reduced from 7th order.
3c	----	1.542211	0.131615	1.215919	----	2,3,5
4a	0.073727	0.771160	----	0.198073	1.116707	6,7,2,3
4b	0.073727	0.771160	0.546060	0.198073	----	6,7,2,3
5a	0.006007	0.043313	0.092802	0.032972	0.015655	6,7,2,3,5

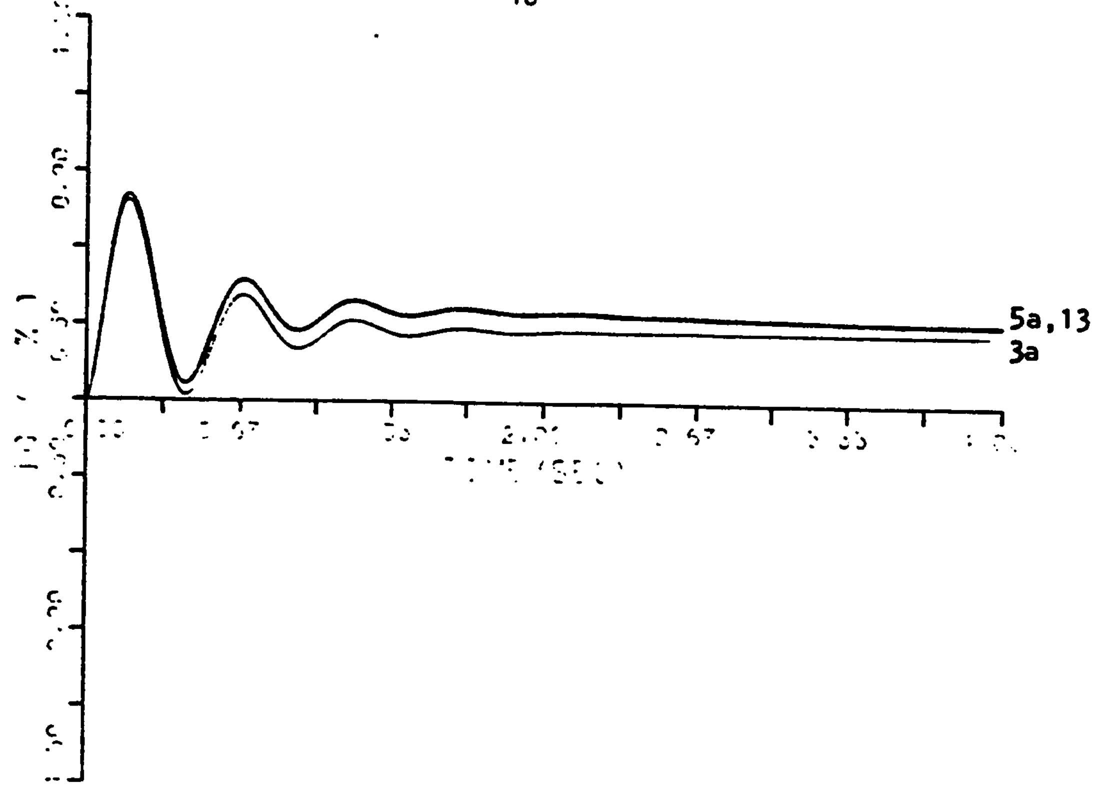


Figure 3.4. I_q response to a 1.0% step change in P_L.

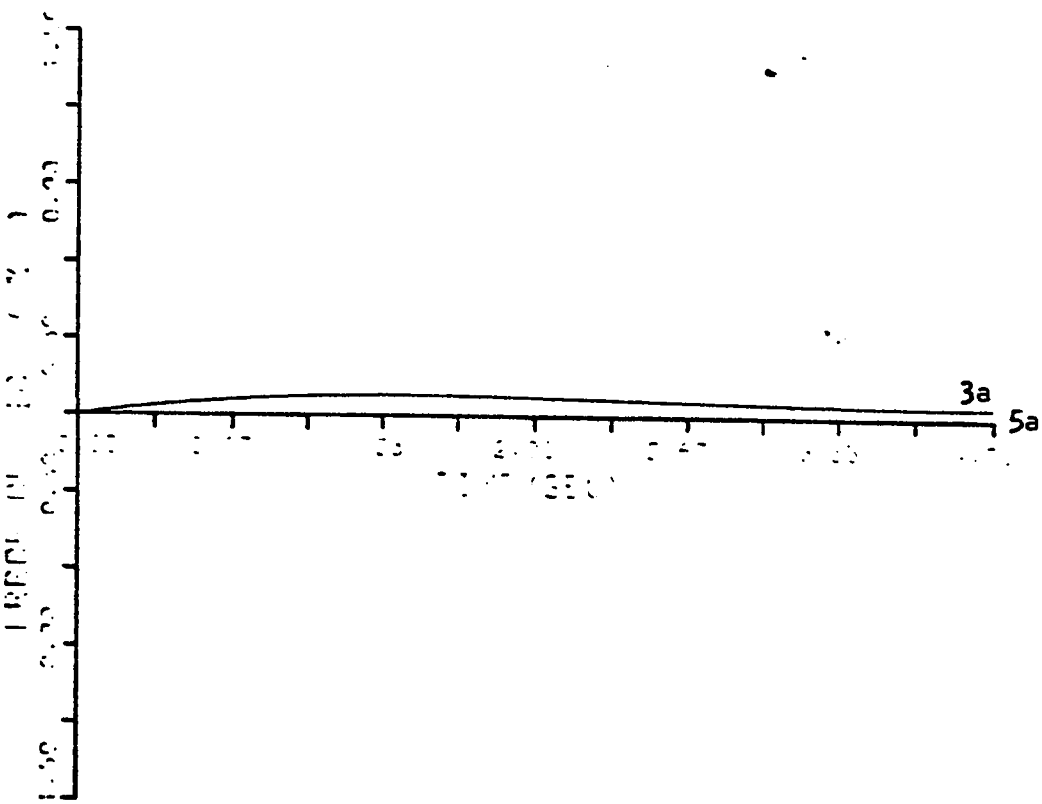


Figure 3.5. I_q error to a 1.0% step change in P_L.

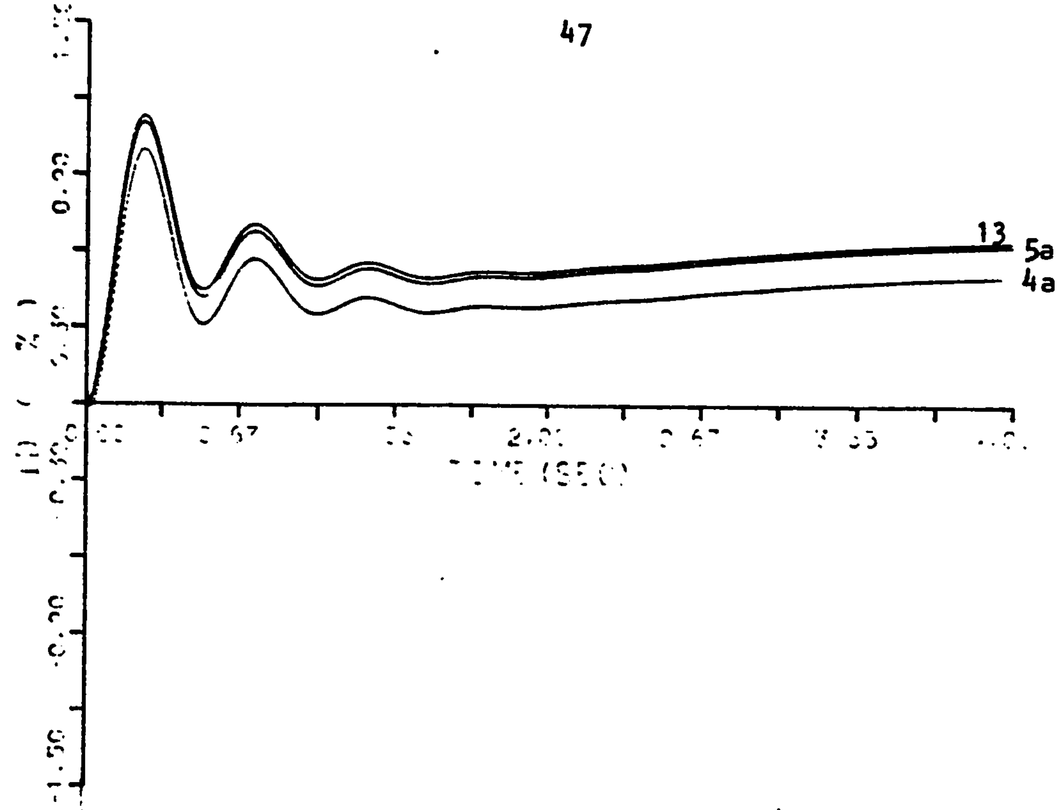


Figure 3.6. I_d response to a 1.0% step change in P_L.

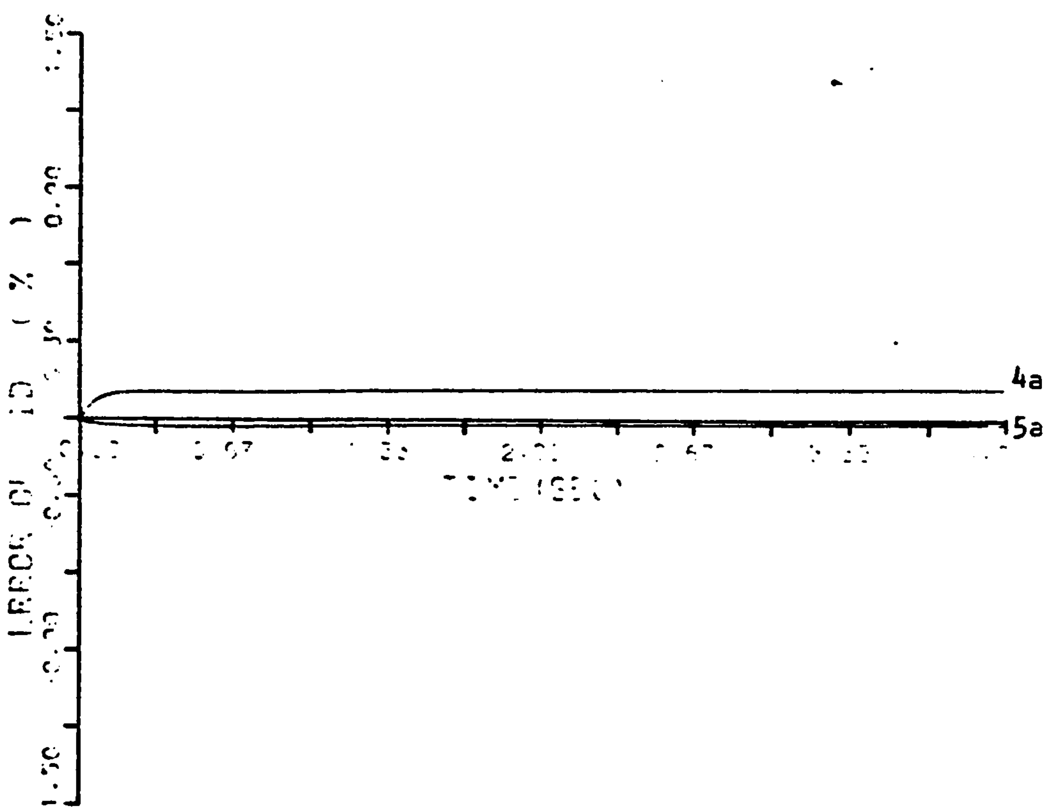


Figure 3.7. I_d error to a 1.0% step change in P_L.

360

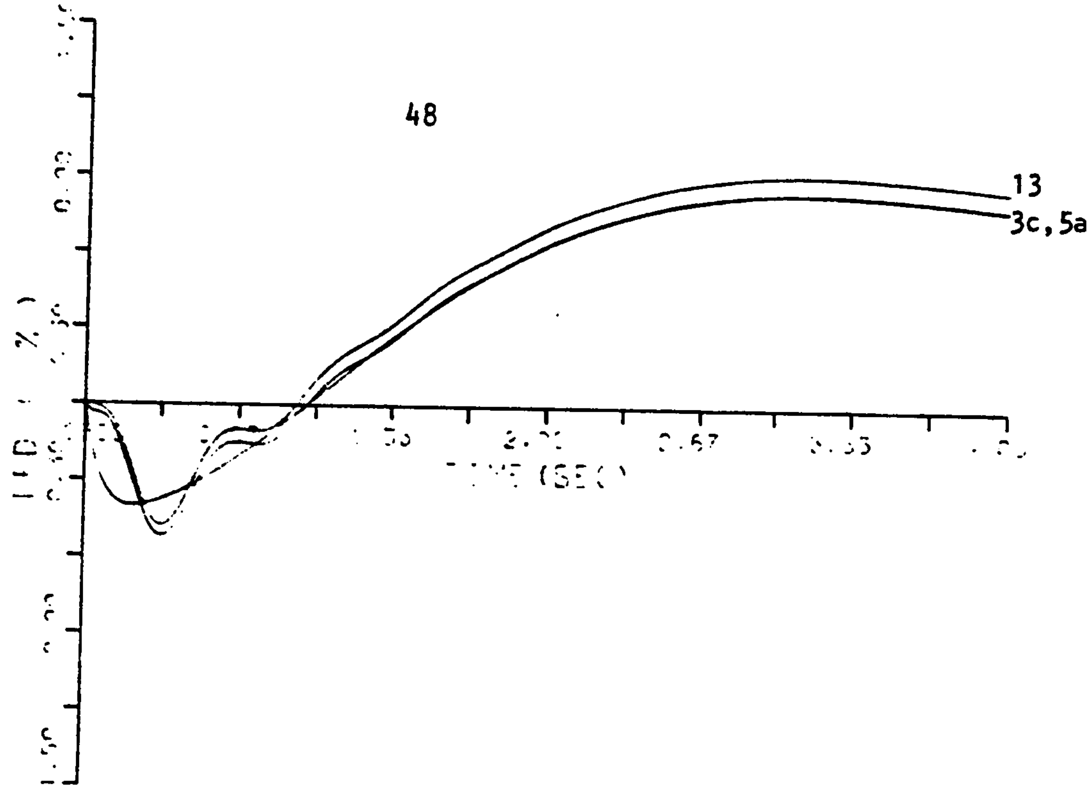


Figure 3.8. Exciter output voltage response to a 1.0% step change in P_L.

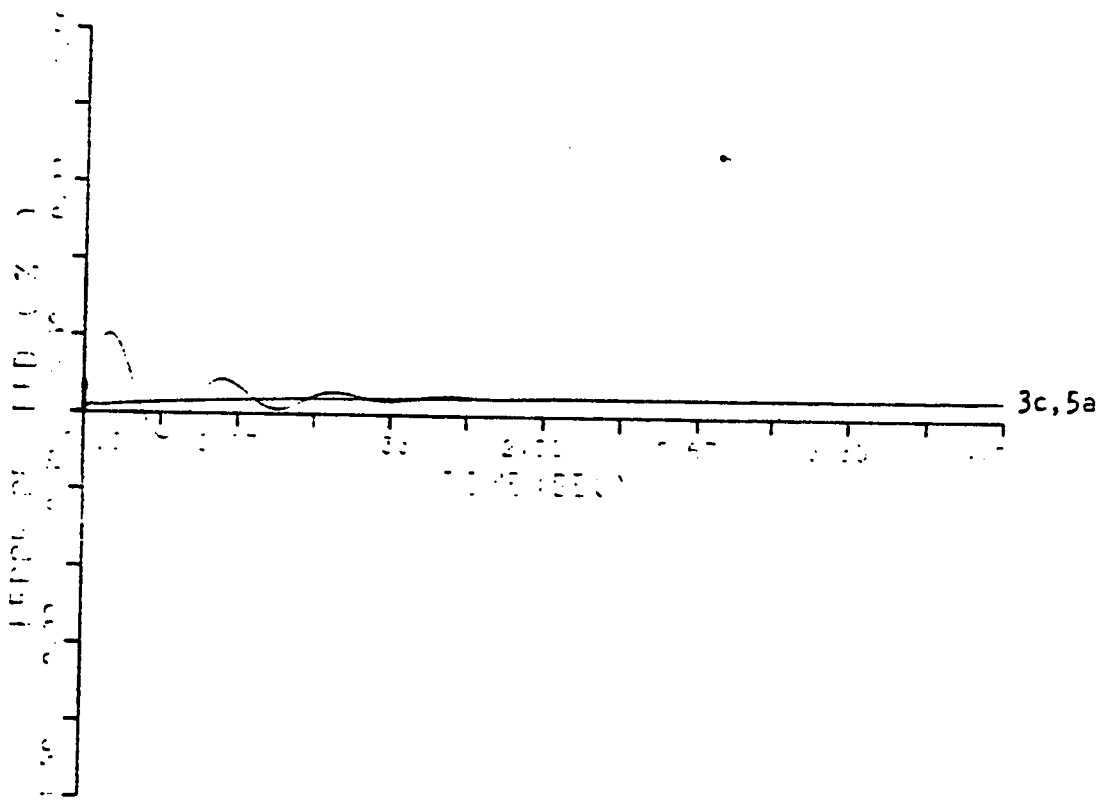


Figure 3.9. Exciter output voltage error to a 1.0% step change in P_L.

360

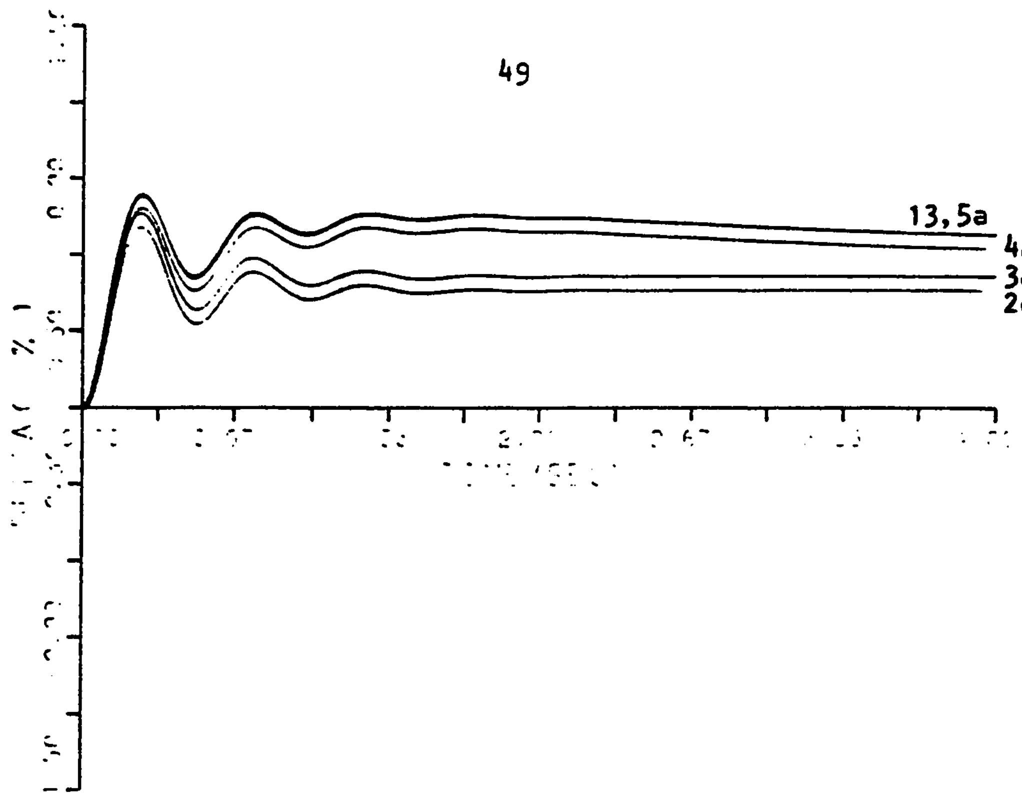


Figure 3.10. Rotor angle response to ~ 1.0% step change in P_L.

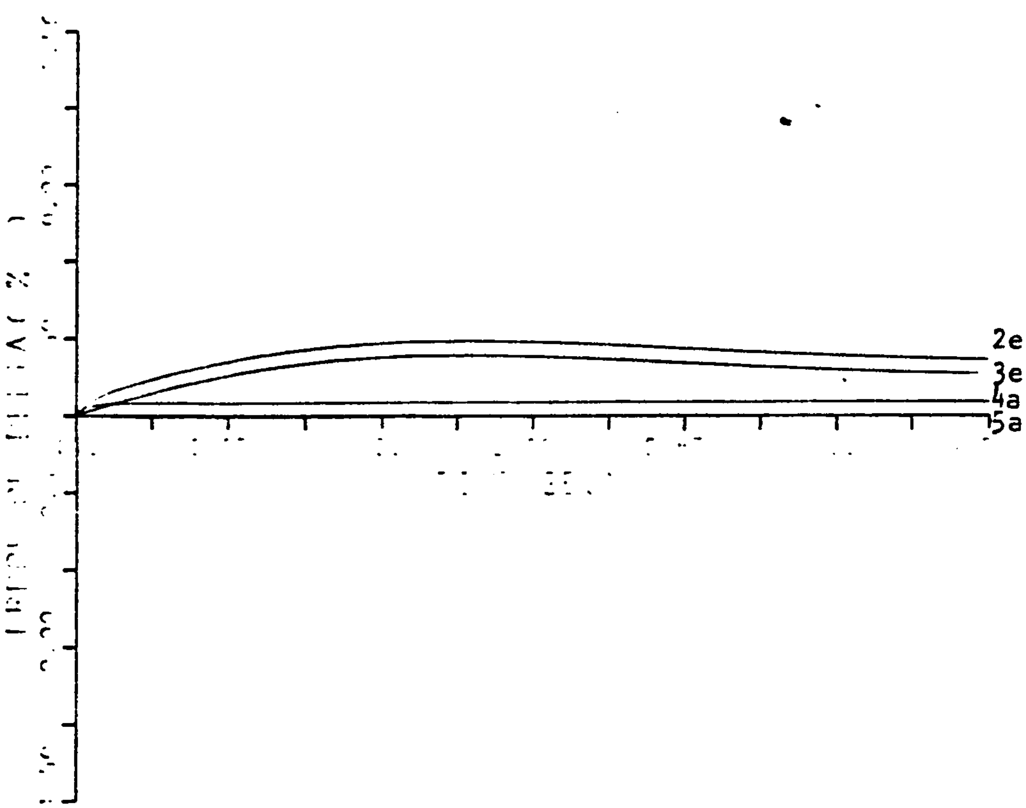


Figure 3.11. Rotor angle error to a 1% step change in P_L.

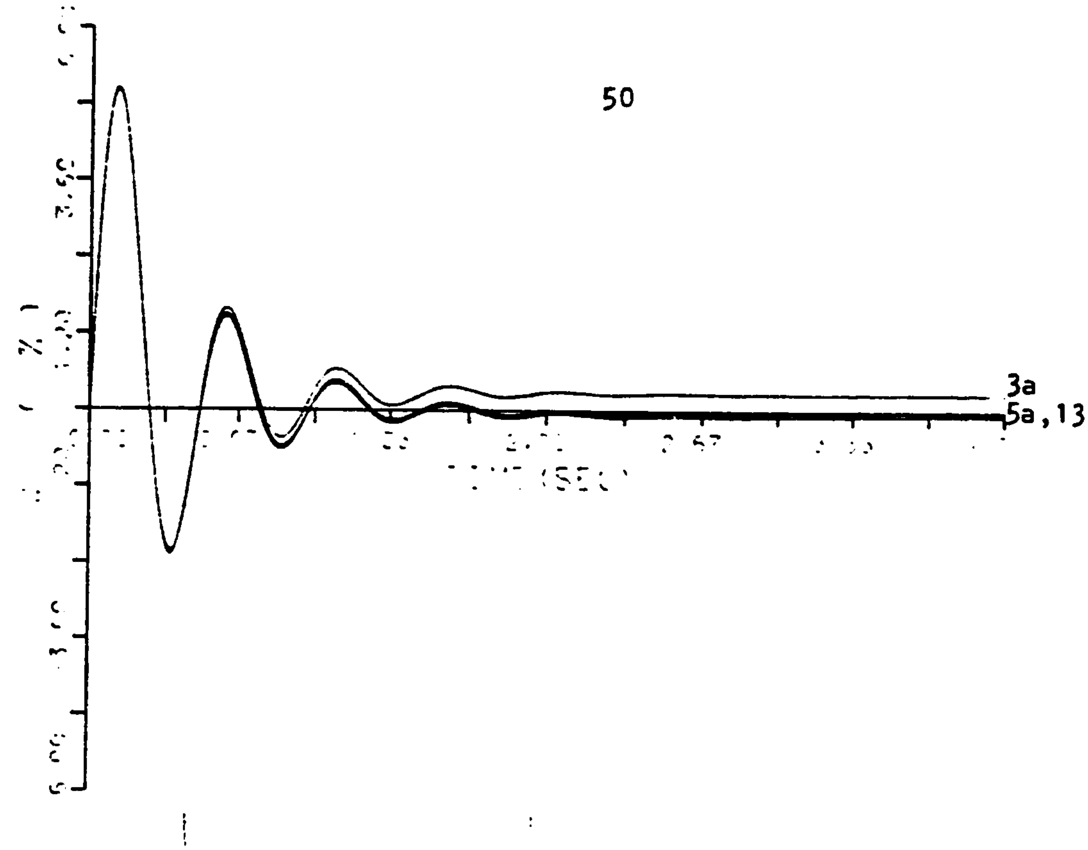


Figure 3.12. Rotor frequency response to a 1.0% step change in P_L.

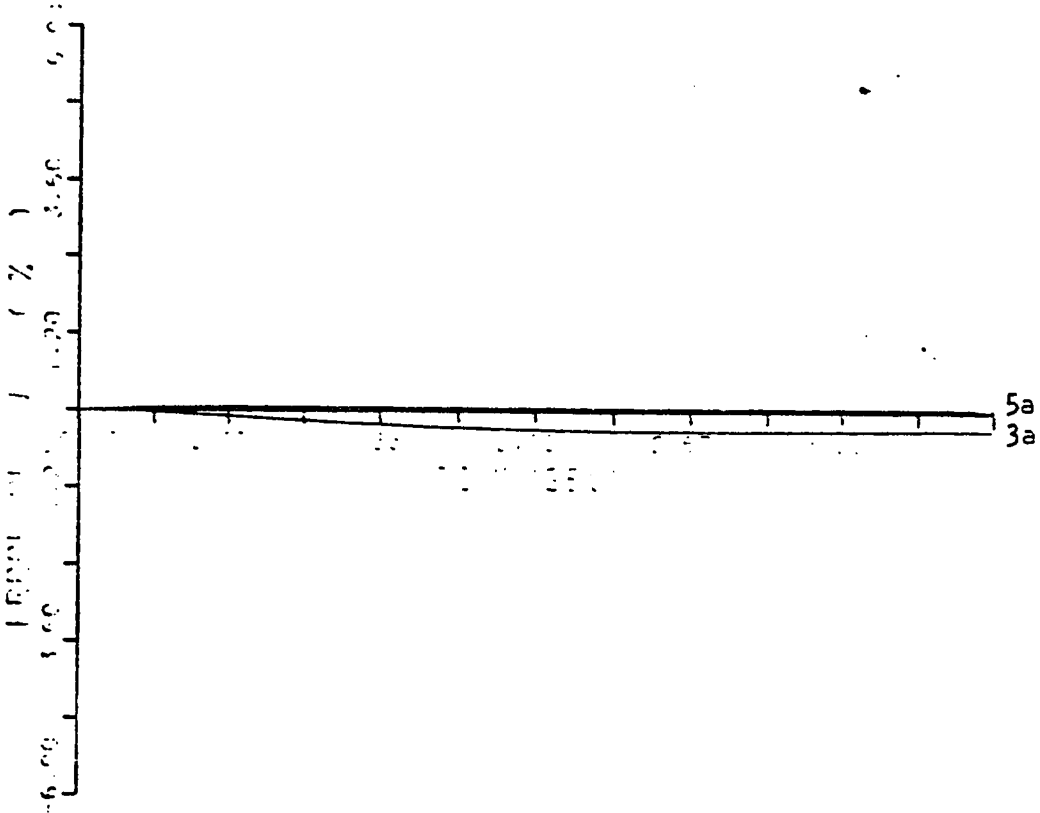


Figure 3.13. Rotor frequency error to a 1.0% step change in P_L.

about few Hz and are poorly damped. From engineering experience modes 2,3 are more related to the interaction between the exciter and the direct axis of the synchronous machine and modes 6,7 are more related to the shaft dynamics. Generally these 7 modes dominate the transient response of the system.

From Table 3.6 the smallest reduced model that can be obtained for rotor angle is a first order from mode 1 which have the highest input-output index or a fourth order from modes 1, 2, 3 and mode 4. A second order from modes 1 and 4 or a third order model from modes 1, 2 and 3 have the same percentage error. We expect this error since mode 2,3 are complex conjugate pair and in the ranking after mode 1 directly. Adding mode 4 to a third order from modes 2,3 and 1 yields results that are practically equivalent to the complete system. If we need more accurate model we can add a second order from modes 6,7 to the fourth order to yield a 6th order while a fifth order by adding modes 6,7 to the previous third order is not good since we ignore mode 4.

The same reduced order model can be applied to the direct axis current, field voltage and the quadrature axis current output to yields a results has the same degree of accuracy except that the error for 1st order for direct axis current and the field voltage are high, as example compare the complete system and the first order model at steady state we find that the first order has a value that approximately closed to the complete model and opposite to it which we expect a degeneration to the result.

For the rotor frequency the order of the ranking are changed and mode 4 is important than mode 2,3, so 1st order or third order give us a high percentage of error while a second order from modes 2,3 or 1,4 are accepted and have the same percentage of error. Still, a fourth order from mode 2,3 and 1,4 yields result that are practically equivalent to the complete system.

Tables 3.7 and 3.8 show the R.M.S. error and the p.u. R.M.S. error of the reduced models which are calculated over a four second period using the error formula derived in chapter 2 respectively. The p.u. R.M.S. error can be used as a criterion to ranking the accurate reduced order models for each output.

The output of the reduced order models are calculated and plotted versus the complete response of 13th order model to a 1.0 %. step change in the governor power, also the error of the reduced order models are plotted down each output over the same studied period in Figs. 3.14 through 3.23.

3.6 CASE 3: THE INPUT IS CONSIDERED TO BE THE CHANGE IN THE SETTING OF THE EXCITER VOLTAGE

Table 3.9 shows the input-output performance index for this case. There are some modes which are not important in the previous cases but have become important in this case. As an example modes 9,10 and mode 8 have become important for the field voltage and the direct axis current. This is because the field is associated with direct axis and according to this any change in the field yields a change in the direct axis dynamic.

TABLE 3.6. Input - Output performance indices to a change in the governor power setting .

Mode	Eigenvalue	Rank	Delta	Rank	Frequncy	Rank	Field voltage	Rank	IQ	Rank	ID
1	-.9680 +j 0.000	1	.6300 + j 0.000	2	-.6090 +j 0.000	1	1.4750 +j 0.000	1	0.2380 +j 0.000	1	1.320 +j 0.000
2	-.6610 +j 1.018	3,4	.1060 + j 0.087	3,4	.0190 -j 0.166	2,3	-.4270 +j 0.615	2,3	0.0500 +j 0.021	3,4	-0.179 -j 0.116
3	-.6610 -j 1.018		.1060 - j 0.087		.0190 +j 0.166		-.4270 -j 0.615		0.0500 -j 0.021		-0.179 +j 0.116
4	-4.137 +j 0.000	2	-.1500 + j 0.000	1	.6220 +j 0.000	4	0.1620 +j 0.000	4	0.0170 +j 0.000	2	-0.329 +j 0.000
5	-15.14 +j 0.000		.0010 + j 0.000		-.0210 +j 0.000	5	-.0110 +j 0.000	5	-.0110 +j 0.000		0.003 +j 0.000
6	-2.574 +j 12.85	5,6	-.0050 + j 0.002	5,6	-.0140 -j 0.076		0.0000 -j 0.004		-.0080 -j 0.005	5,6	-.009 +j 0.003
7	-2.574 -j 12.85		-.0050 - j 0.002		-.0140 +j 0.076		0.0000 +j 0.004		-.0080 +j 0.005		-.009 -j 0.003
8	-38.25 +j 0.000		.0000 + j 0.000		0.0000 +j 0.000		0.0140 +j 0.000		0.0000 +j 0.000		0.000 +j 0.000
9	-26.24 +j 39.78		.0000 + j 0.000		0.0000 +j 0.000		0.0020 +j 0.000		0.0000 +j 0.000		0.000 +j 0.000
10	-26.24 -j 39.78		.0000 + j 0.000		0.0000 +j 0.000		0.0020 +j 0.000		0.0000 +j 0.000		0.000 +j 0.000
11	-26.75 +j376.03		.0000 + j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.000 +j 0.000
12	-26.75 -j376.03		.0000 + j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.000 +j 0.000
13	-999.99-j 0.000		.0000 + j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.000 +j 0.000

Table 3.7. R.M.S % Error of reduced models of 13th order system for 1. % change in the governor power.

ORDER OF MODEL	DELTA	FREQUENCY	FIELD VOLTAGE	ID	IQ	PRESERVED EIGENVALUES
1a	0.074657	----	----	----	----	1
1b	----	0.641931	----	----	----	1
1c	----	----	----	----	0.080603	1
1d	----	----	----	0.625747	----	1
1e	----	----	0.889085	-----	----	1
2a	0.171567	0.075313	----	----	----	1,4
2b	----	----	----	0.321618	0.066143	1,4
2c	----	----	1.041006	0.321618	----	1,4
2g	----	----	1.305279	0.725711	----	2,3
2h	0.349762	0.124008	----	----	----	2,3
3e	0.152793	----	----	0.327926	0.012156	1,2,3
3b	----	0.544339	0.143338	----	0.012156	1,2,3
4e	0.009572	0.056538	0.011201	0.013954	----	1,2,3,4
4f	0.009572	----	0.011201	0.013954	0.027533	1,2,3,4
5e	0.010958	0.038327	0.001384	0.017320	0.017092	1,2,3,4,5

Table 3.8. P.U. R.M.S % Error of reduced models of 13th order system for 1. % change in the governor power.

ORDER OF MODEL	DELTA	FREQUENCY	FIELD VOLTAGE	ID	IQ	PRESERVED EIGENVALUES
1a	0.141213	----	----	----	----	1
1b	----	2.877358	----	----	----	1
1c	----	----	----	----	0.301665	1
1d	----	----	----	1.496420	----	1
1e	----	----	2.230142	-----	----	1
2a	0.324533	0.337584	----	----	----	1,4
2b	----	----	----	0.769130	0.247545	1,4
2c	----	----	2.611215	0.769130	----	1,4
2g	----	----	3.274097	1.735485	----	2,3
2h	0.661573	0.555850	----	----	----	2,3
3a	0.289008	----	----	0.430150	0.045495	1,2,3
3b	----	2.439874	0.359528	----	0.045495	1,2,3
4e	0.018106	0.253424	0.028097	0.033371	----	1,2,3,4
4f	0.018106	----	0.028097	0.033371	0.103045	1,2,3,4
5e	0.020727	0.171790	0.003471	0.041421	0.063969	1,2,3,4,5

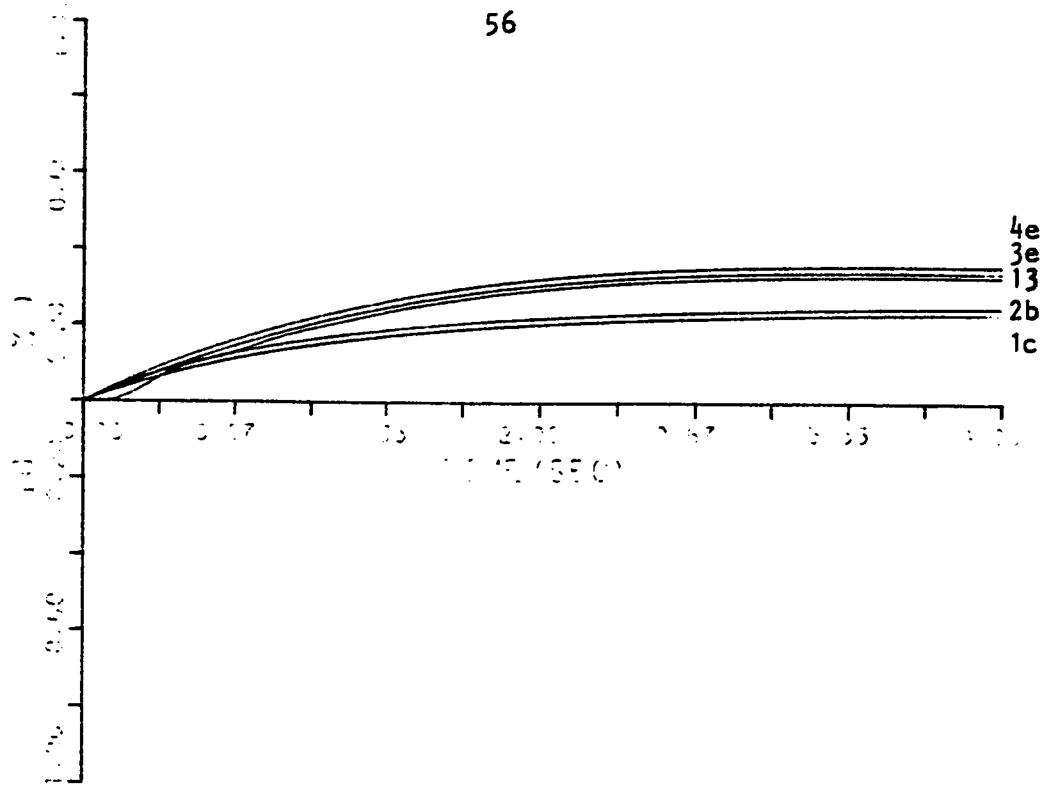


Figure 3.14. I_q response to a 1.0% step change in P_c.

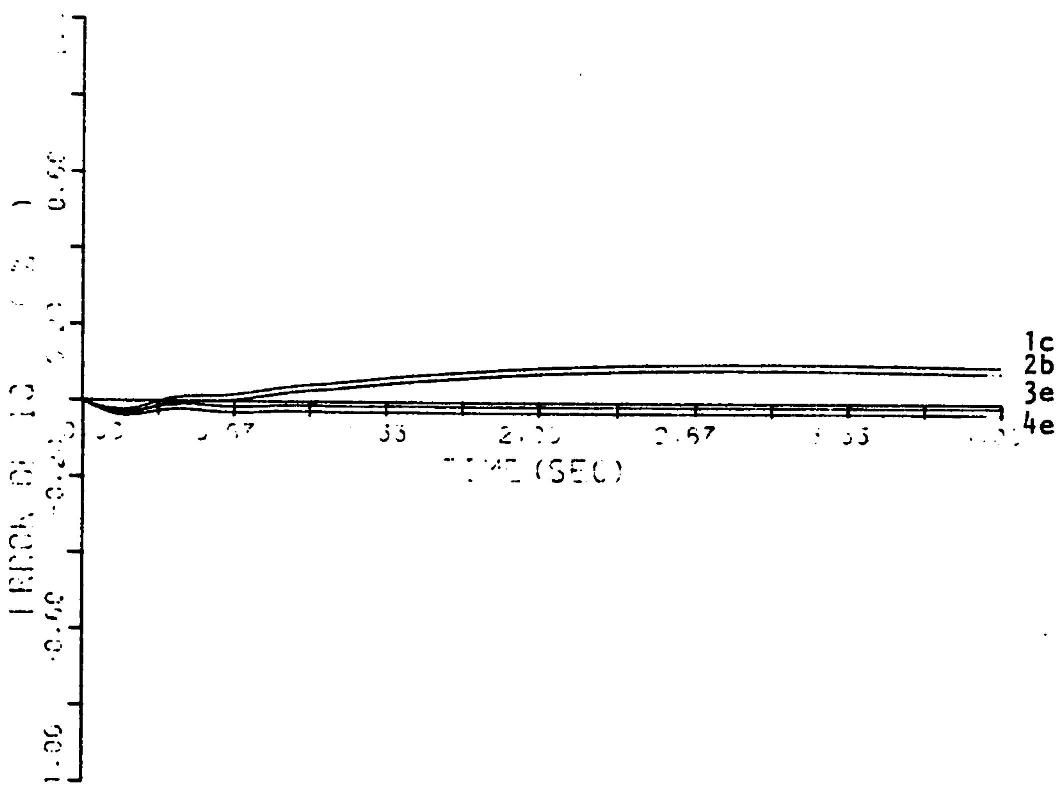


Figure 3.15. I_q error to a 1.0% step change in P_c.

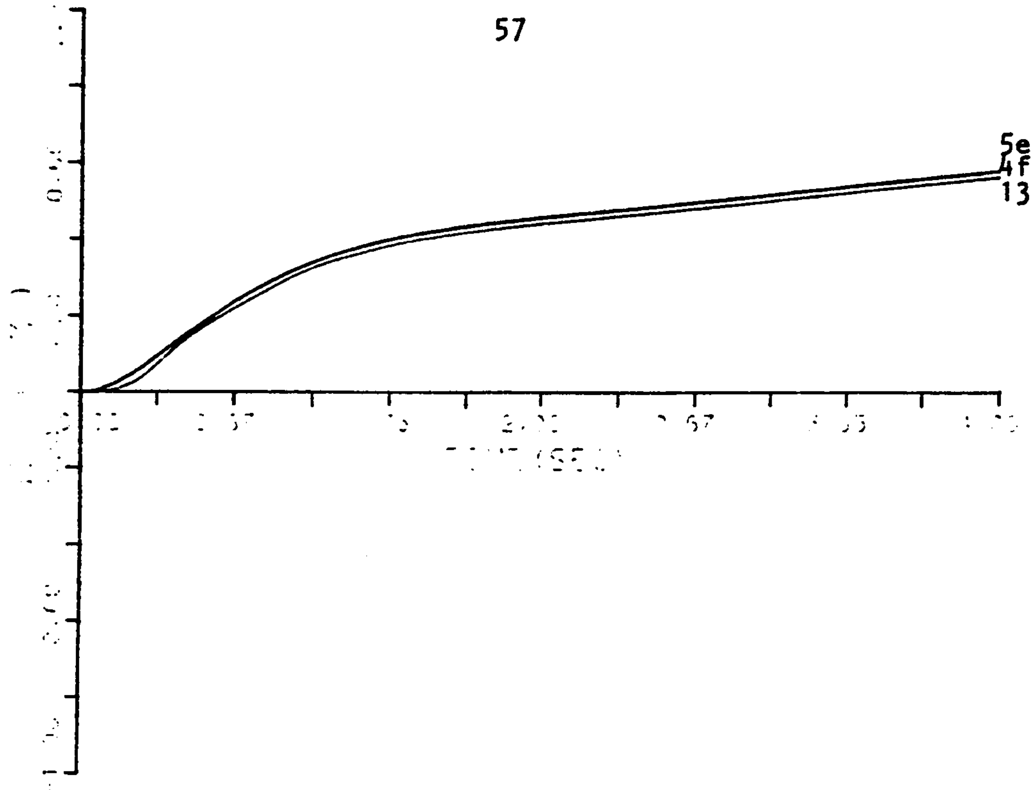


Figure 3.16. I_d response to a 1.0% step change in P_c.

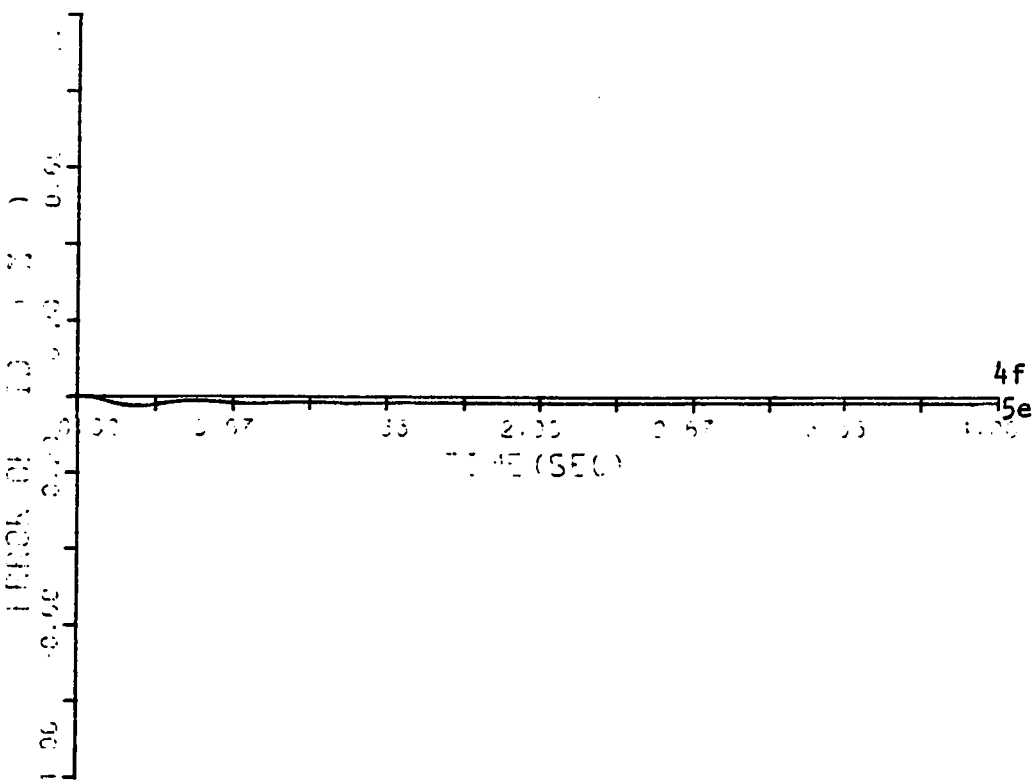


Figure 3.17. I_d error to a 1.0% step change in P_c.

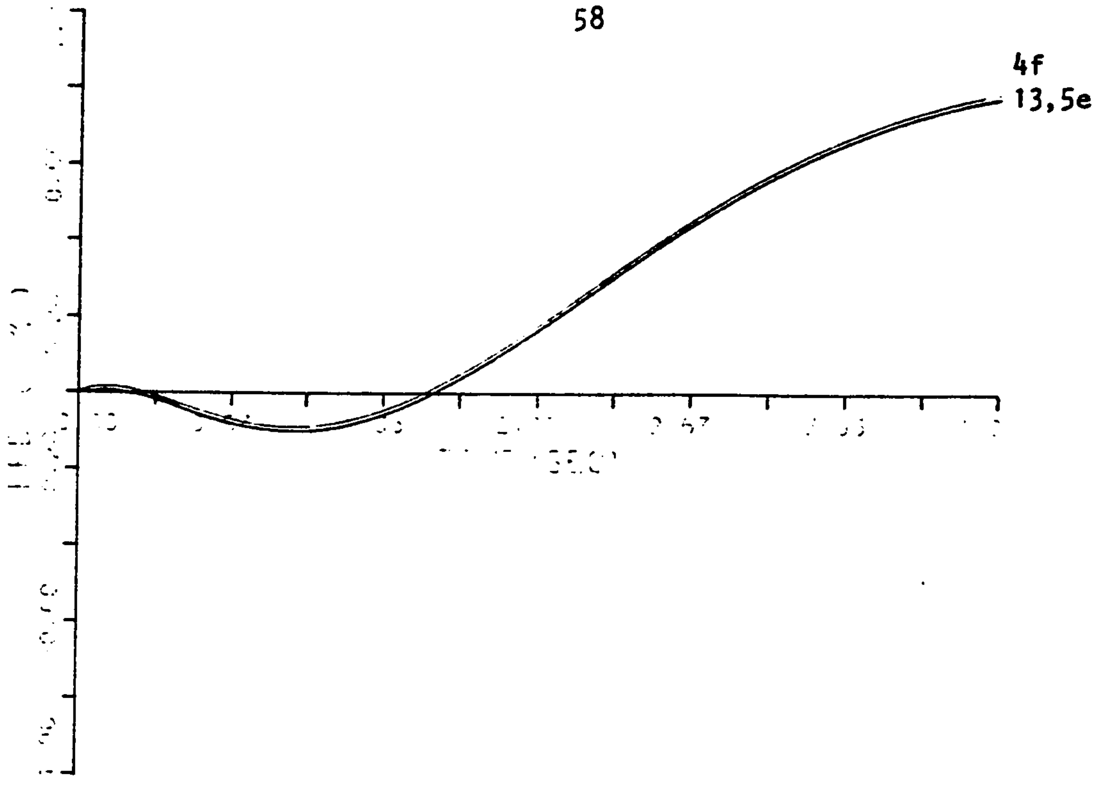


Figure 3.18. Exciter output voltage response to a 1% step change in P_c.

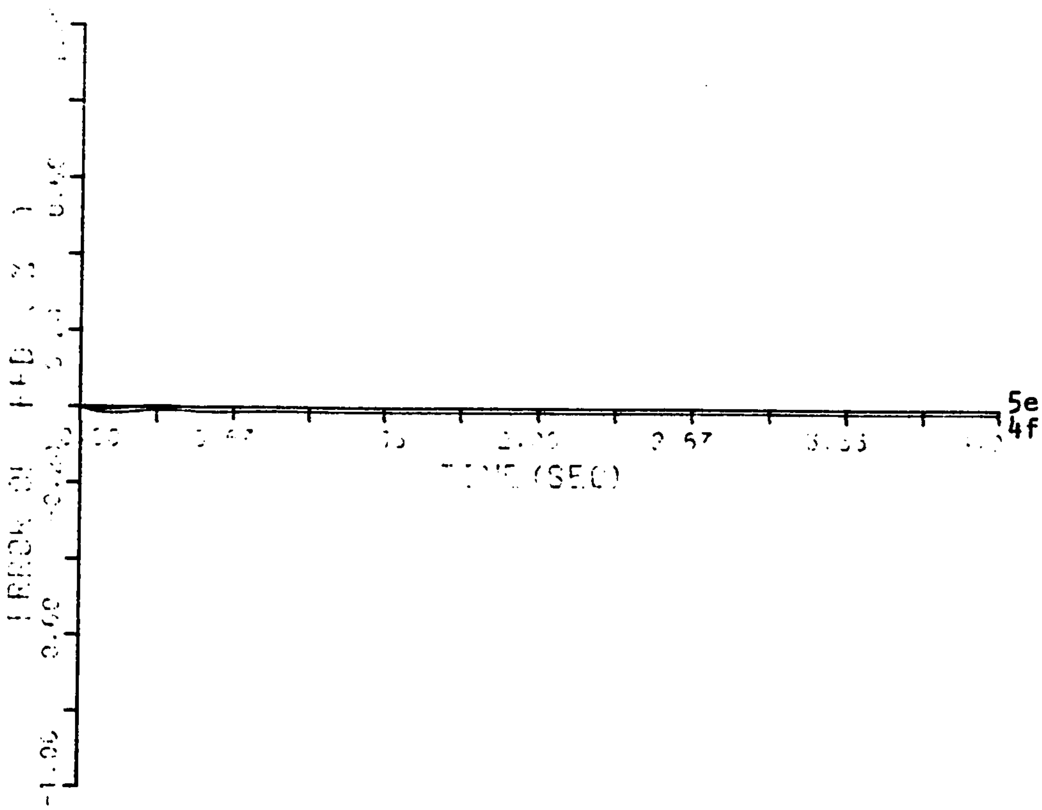


Figure 3.19. Exciter output voltage error to a 1.0% step change in P_c.

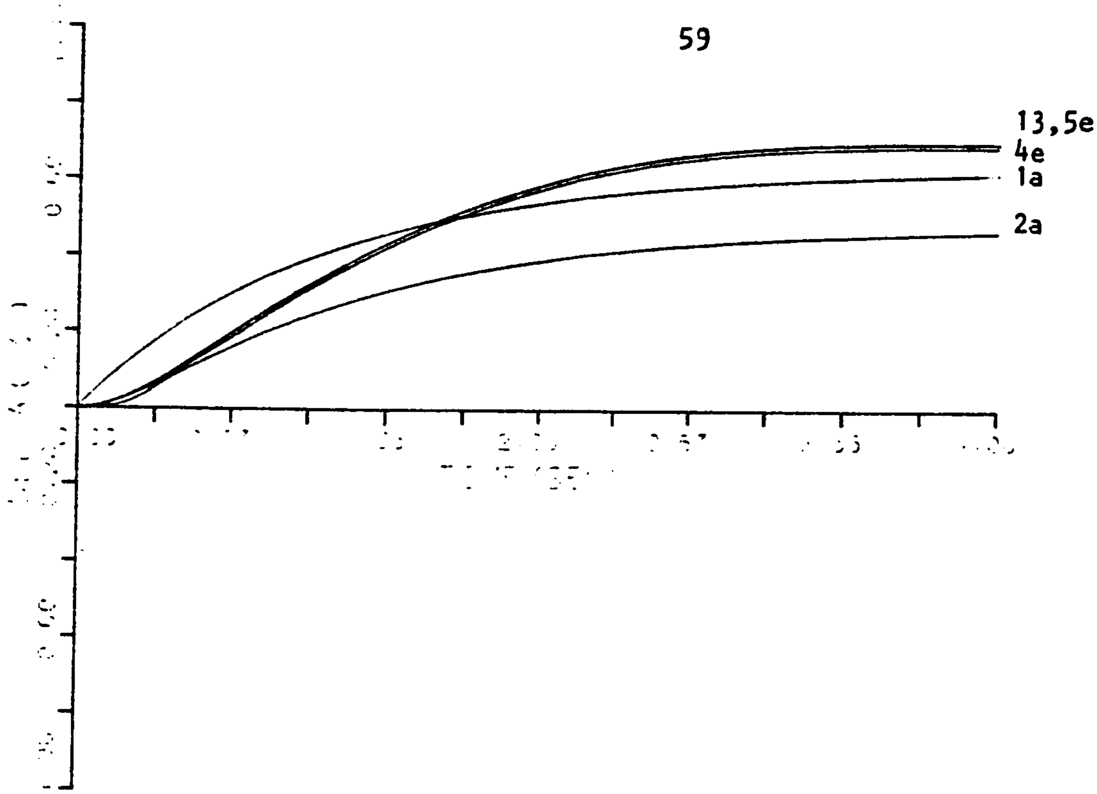


Figure 3.20. Rotor angle response to a 1.0% step change in P_c.

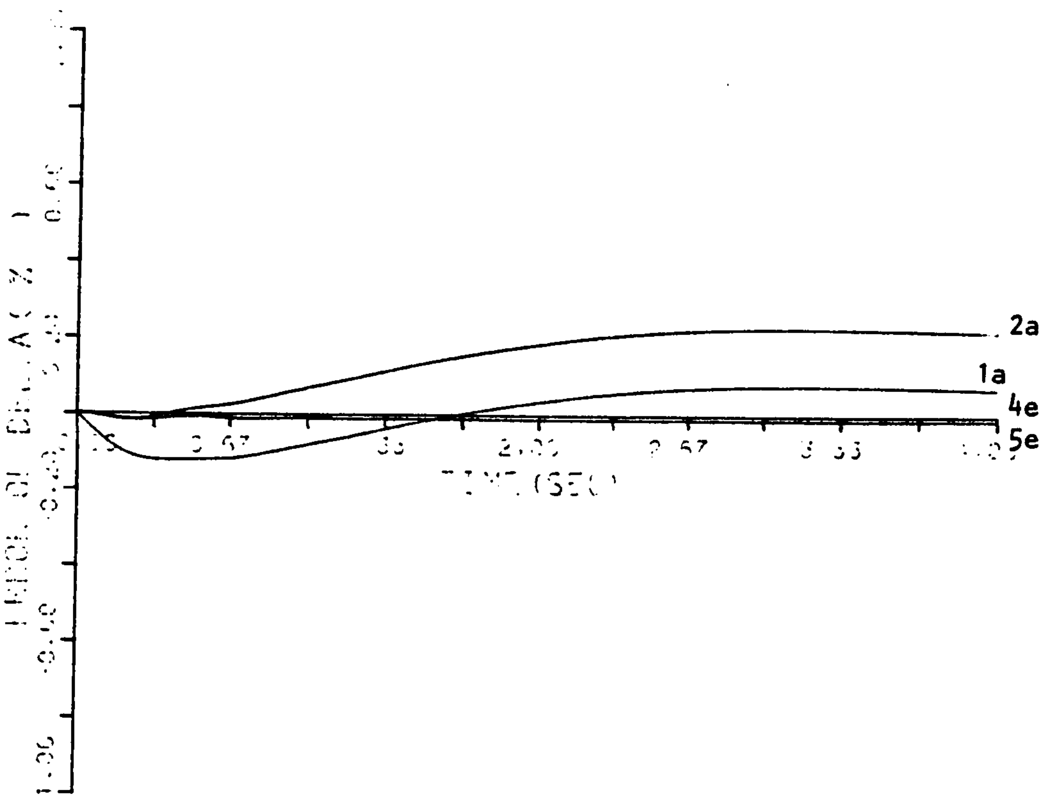


Figure 3.21. Rotor angle error to a 1.0% step change in P_c.

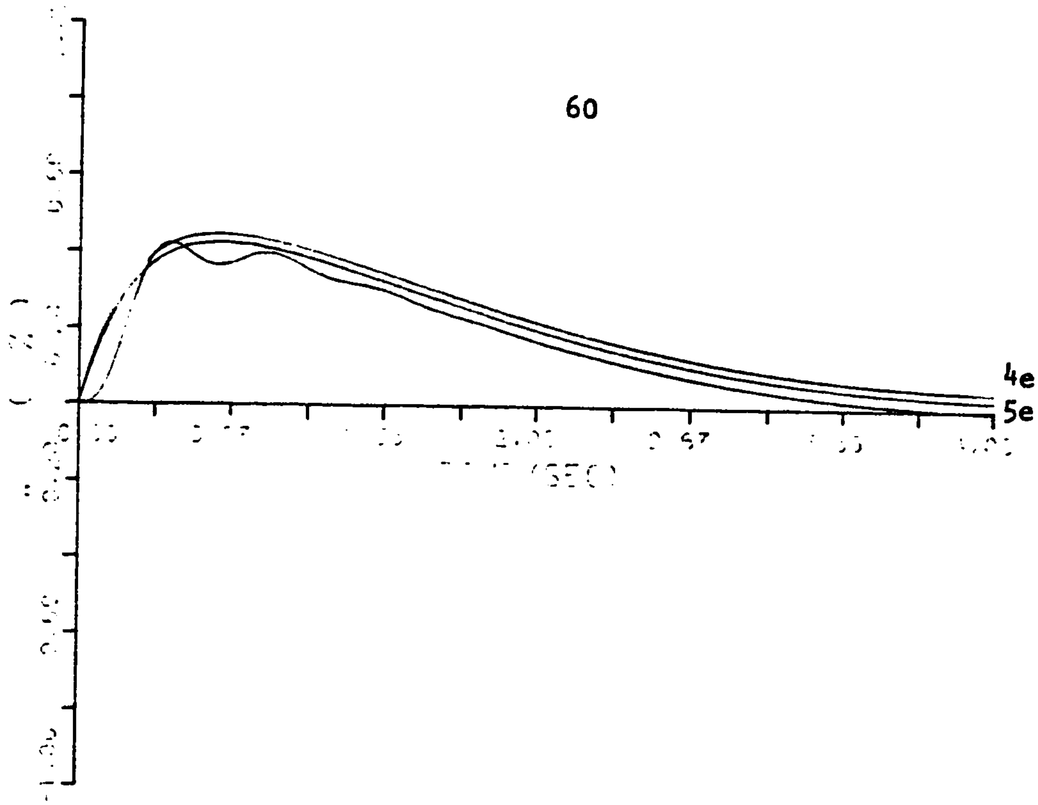


Figure 3.22. Rotor frequency response to a 1.0% step change in P_c.

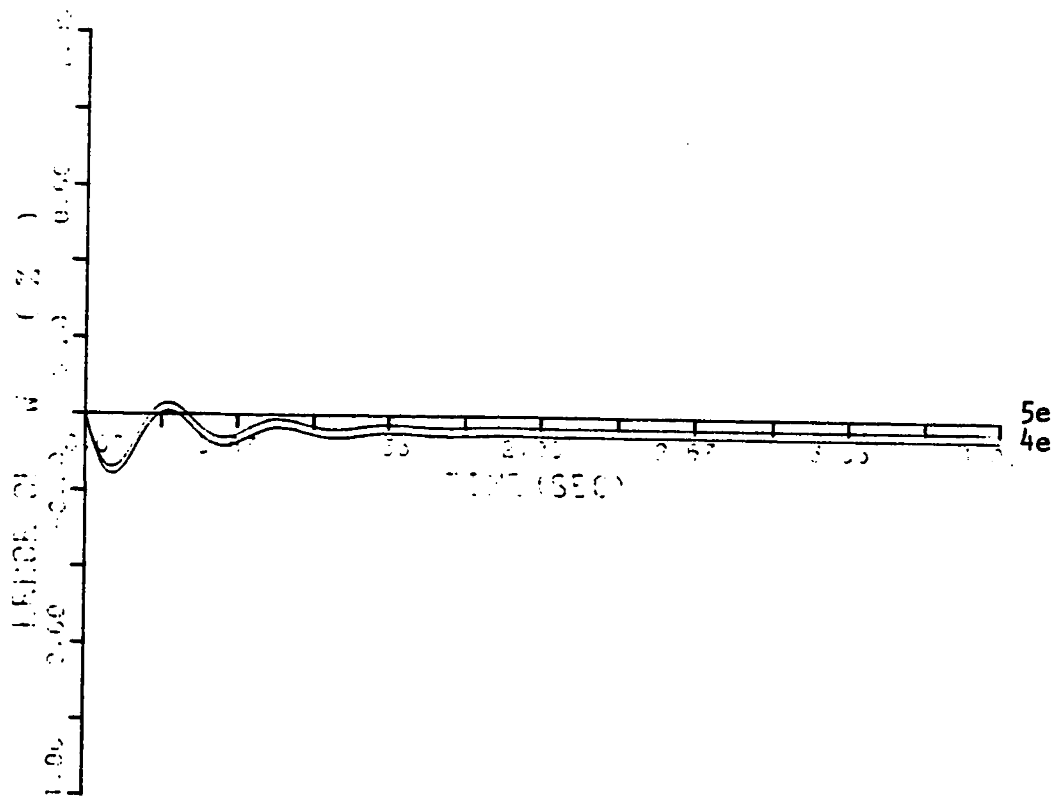


Figure 3.23. Rotor frequency error to a 1.0% step change in P_c.

The modes which are found to be dominant in the complete system are the same previous modes adding to them the complex conjugate pair of modes 9, 10 and the real pole of mode 8.

Modes 2, 3 play an important point in the system. This is clear from the T-factor and according to this, a second order from modes 2,3 for all the output can be calculated but remember that the accuracy is not satisfied (i.e. high error expected). The modes which come after modes 2,3 in the ranking are different from output to output. This implies that we have more than one third order model.

For the direct axis current and the rotor angle a third order from modes 2,3 and 1 give accurate result while we find for the quadrature axis current a third order model from modes 2,3 and 5 is more accurate. For the rotor frequency and the field voltage output a third order can be obtained using the previous modes with permissible error. A fourth order from modes 6,7 and 2,3 is accurate for the rotor frequency output and the rotor angle, while a fourth order from modes 2,3 and 9,10 is accurate for the field voltage and the direct axis current.

The fifth order model takes different situation because the modes do not have the same ranking in the input-output performance indices. We can summarize the accurate model for each output as follows:

1- A fifth order from modes 2,3,6,7 and 5 are accurate for rotor frequency and the quadrature axis current.

2- A fifth order from modes 2,3,6,7 and 1 are accurate for rotor angle.

3- A fifth order from modes 2,3,9,10 and 1 are accurate for field voltage and the direct axis current.

The error obtained for fifth order is small enough which means that a fifth order is practically equivalent to the complete system.

Tables 3.10 and 3.11 show the R.M.S. error and the p.u. R.M.S. error of the reduced models which are calculated over a four second period using the error formulae derived in chapter 2 respectively. The p.u. R.M.S. error can be used as a criterion in ranking the accurate reduced order models for each output.

The output of the reduced order models are calculated and plotted versus the complete response of 13th order model for a 1.0% step change in the exciter output voltage, also the error of the reduced order models are

TABLE 3.9. Input - Output performance indices to a change in the exciter reference voltge.

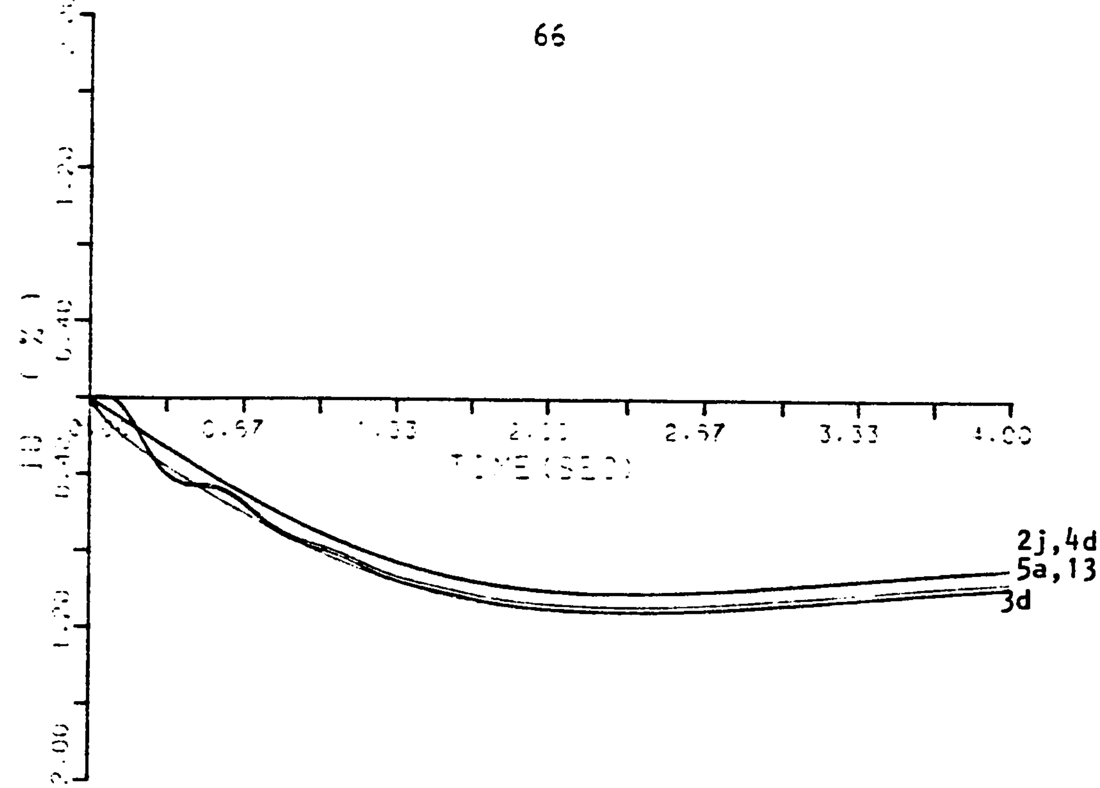
Mode	Eigenvalue	Rank	Delta	Rank	Frequency	Rank	Field voltage	Rank	IQ	Rank	ID
1	-.9680 +j 0.000	5	-.0530 + j 0.000	6	.0520 +j 0.000	6	-.1250 +j 0.000		-.0200 +j 0.000	3	-.112 +j 0.000
2	-.6610 +j 1.018	1,2	-1.0840 - j 0.087	1,2	.6180 +j 1.167	1,2	0.0460 -j 5.929	1,2	-.4250 +j 0.085	1,2	1.689 -j 0.035
3	-.6610 -j 1.018		-1.0840 + j 0.087		.6180 -j 1.167		0.0460 +j 5.929		-.4250 -j 0.085		1.689 +j 0.035
4	-4.137 +j 0.000		-.0090 + j 0.000		.0350 +j 0.000		0.0090 +j 0.000		0.0010 +j 0.000		-0.019 +j 0.000
5	-15.14 +j 0.000	6	.0140 + j 0.000	5	-.2080 +j 0.000		-.1070 +j 0.000	3	-.1030 +j 0.000		0.033 +j 0.000
6	-2.574 +j 12.85	3,4	.0480 + j 0.028	3,4	-.4790 +j 0.551		-.0320 +j 0.027	4,5	0.0150 +j 0.089	4,5	0.073 +j 0.046
7	-2.574 -j 12.85		.0480 - j 0.028		-.4790 -j 0.551		-.0320 -j 0.027		0.0150 -j 0.089		0.073 -j 0.046
8	-38.25 +j 0.000		.0020 + j 0.000		-.0900 +j 0.000	5	0.1450 +j 0.000		-.0010 +j 0.000		-.082 +j 0.000
9	-26.24 +j 39.78		.0000 + j 0.001		-.0340 -j 0.025	3,4	3.6290 -j 2.601		-.0020 +j 0.001		-.041 +j 0.014
10	-26.24 -j 39.78		.0000 - j 0.001		-.0340 +j 0.025		3.6290 +j 2.601		-.0020 -j 0.001		-.041 -j 0.014
11	-26.75 +j376.03		.0000 + j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.000 +j 0.000
12	-26.75 -j376.03		.0000 + j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.000 +j 0.000
13	-999.99-j 0.000		.0000 + j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.0000 +j 0.000		0.000 +j 0.000

Table 3.10. R.M.S % Error of reduced models of 13th order system for 1. % change in the exciter reference voltage

ORDER OF MODEL	DELTA	FREQUENCY	FIELD VOLTAGE	ID	IQ	PRESERVED EIGENVALUES
2g	----	----	7.126379	0.094063	----	2,3
2h	0.069175	1.249743	----	----	----	2,3
2j	0.069175	----	----	----	0.094361	2,3
3d	----	1.046726	7.231443	----	0.031627	2,3,5
3e	0.106488	----	----	0.022960	0.079484	2,3,1
4a	0.035024	0.290478	----	0.234424	0.122646	2,3,6,7
4c	0.104183	1.200500	7.075185	0.083812	----	2,3,1,8
4d	----	1.183825	0.115609	0.040943	0.089734	2,3,9,10
5a	0.047909	0.086564	7.295377	0.267282	0.020926	2,3,6,7,5
5b	0.037178	0.201396	7.045604	0.153692	0.121434	2,3,6,7,8
5c	0.106585	1.222247	0.022341	0.084692	0.074931	2,3,9,10,1
5d	0.007796	0.328905	7.283787	0.148922	0.107380	2,3,6,7,1

Table 3.11. P.U. R.M.S % Error of reduced models of 13th order system for 1. % change in the exciter reference voltage.

ORDER OF MODEL	DELTA	FREQUENCY	FIELD VOLTAGE	ID	IQ	PRESERVED EIGENVALUES
2g	----	----	0.738214	0.029962	----	2,3
2h	0.034885	1.374234	----	----	----	2,3
2J	0.034885	----	----	----	0.101750	2,3
3d	----	1.151002	0.749097	----	0.034101	2,3,5
3e	0.053701	----	----	0.007313	0.085708	2,3,1
4a	0.017662	0.319414	----	0.074672	0.132250	2,3,6,7
4c	0.0525539	1.320089	0.732909	0.026697	----	2,3,1,8
4d	----	1.301751	0.011976	0.013042	0.096761	2,3,9,10
5a	0.024160	0.095188	0.755719	0.085139	0.022565	2,3,6,7,5
5b	0.018749	0.221458	0.729846	0.048955	0.130943	2,3,6,7,8
5c	0.053750	1.344001	0.002314	0.026977	0.080799	2,3,9,10,1
5d	0.003931	0.361671	0.754516	0.047435	0.115788	2,3,6,7,1



Figu e 3.24. I_q response to a 1.0% step change in V_c.

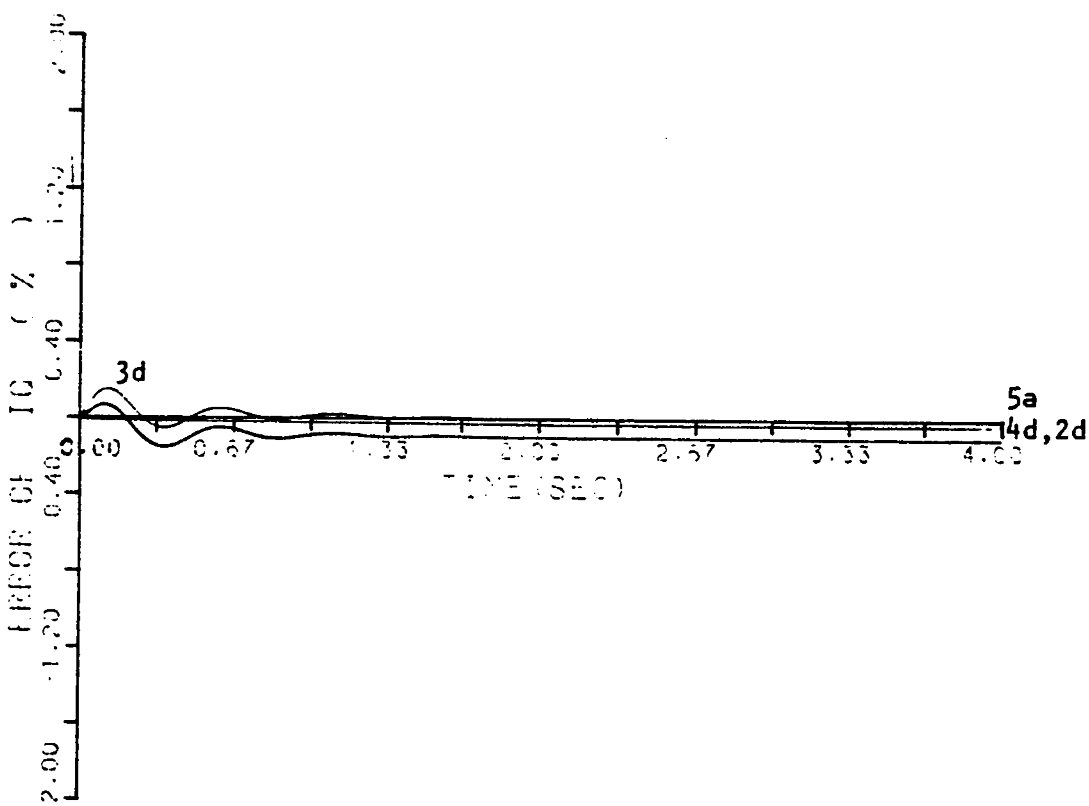


Figure 3.25. I_q error to a 1.0% step change in V_c.

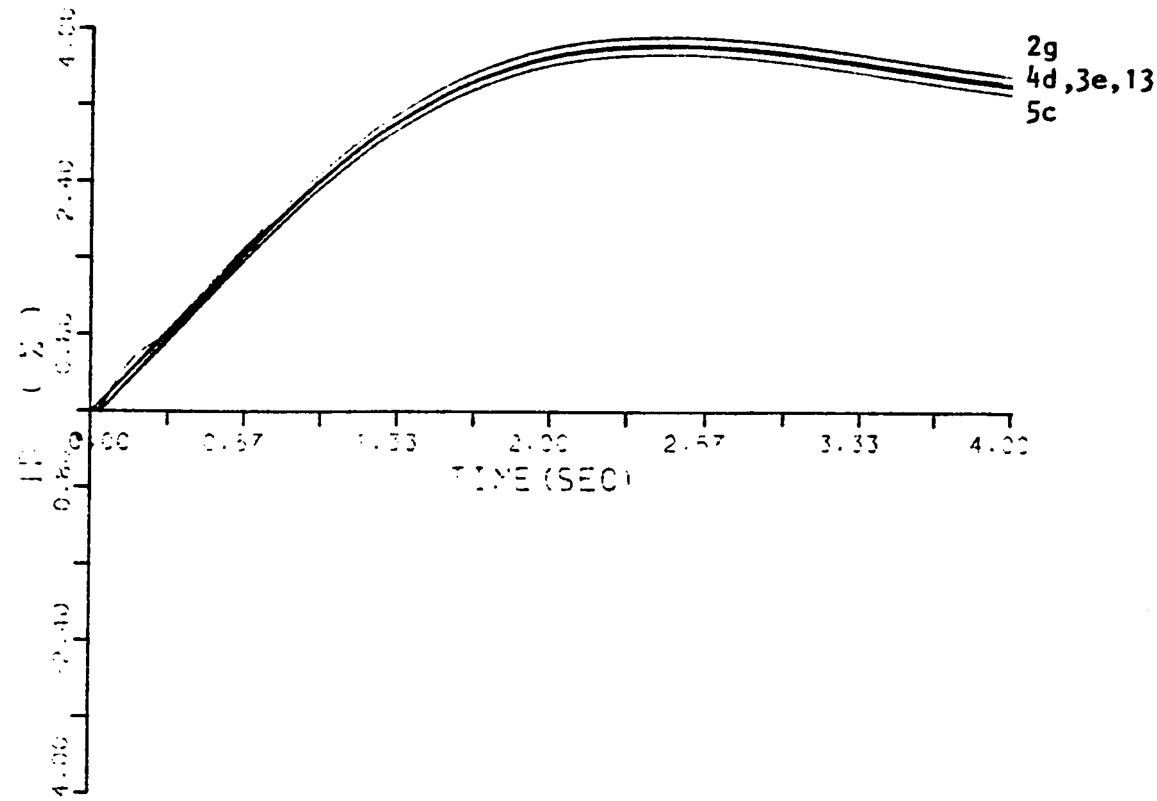


Figure 3.26. I_d response to a 1.0 step change in V_c.

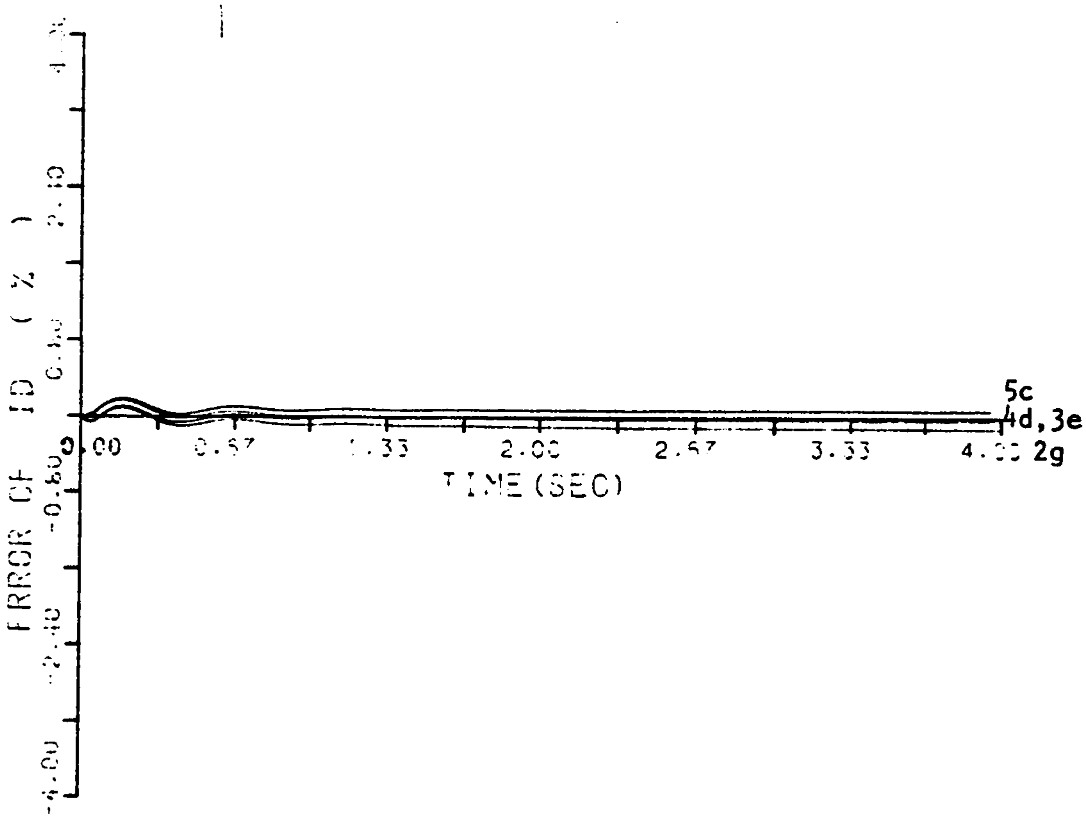


Figure 3.27. I_d error to a 1.0% step change in V_c.

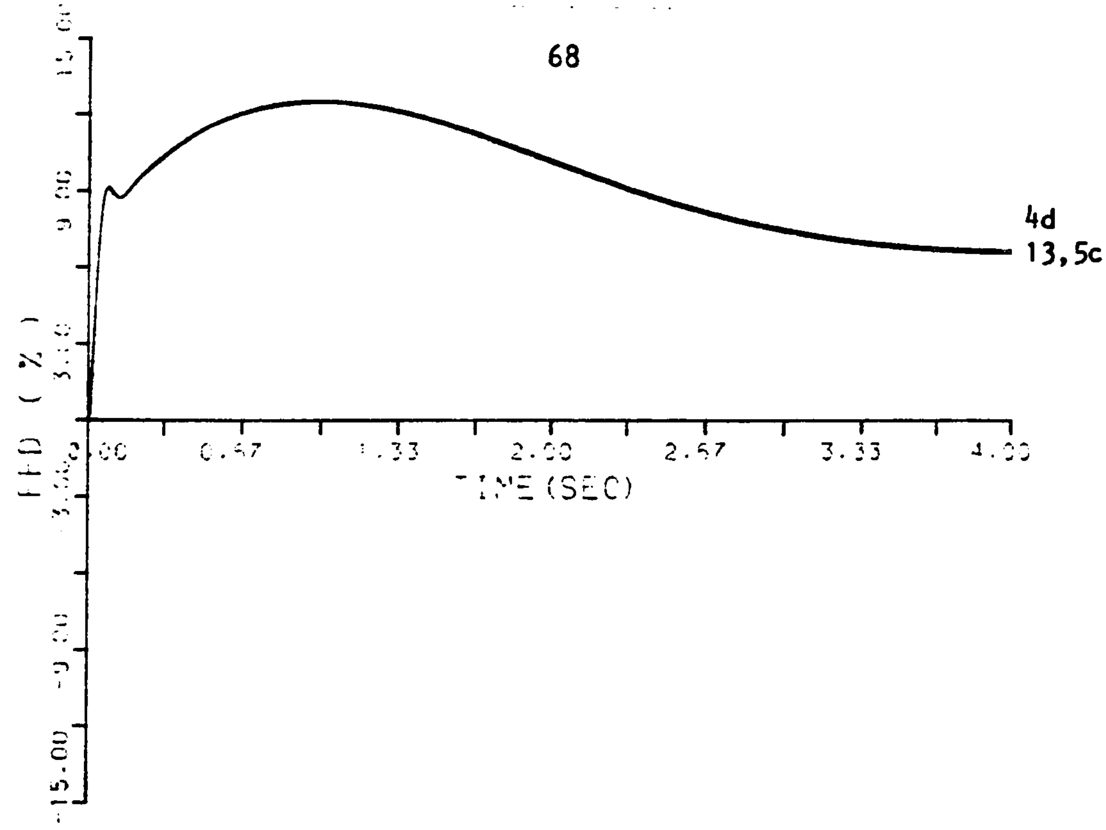


Figure 3.28. Exciter output voltage response to a 1.0% step change in V_c.

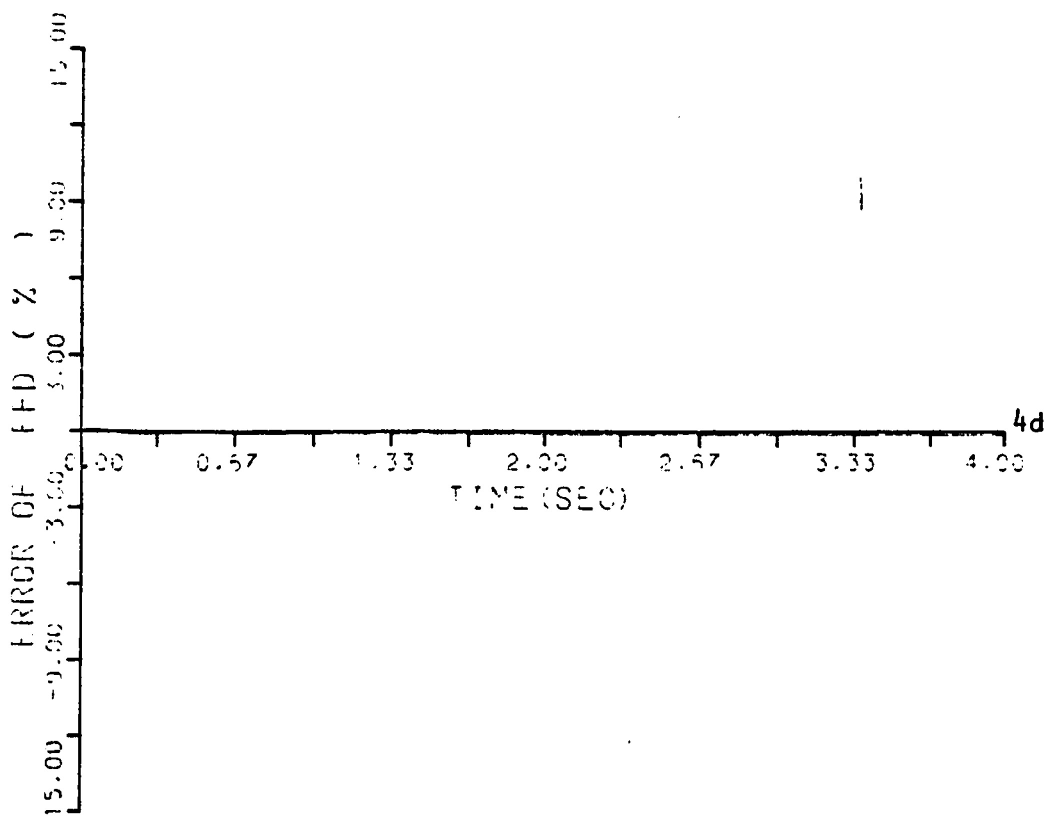


Figure 3.29. Exciter output voltage error to a 1.0% step change in V_c.

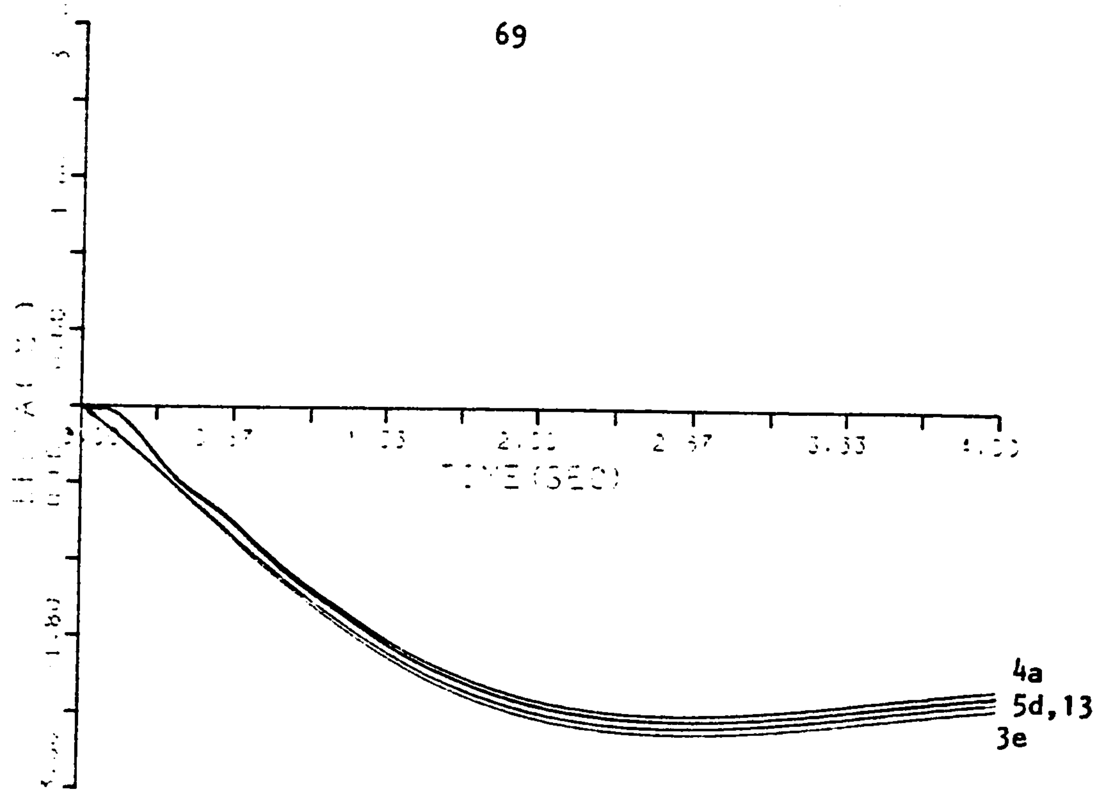


Figure 3.30. Rotor angle response to a 1.0% step change in V_c.

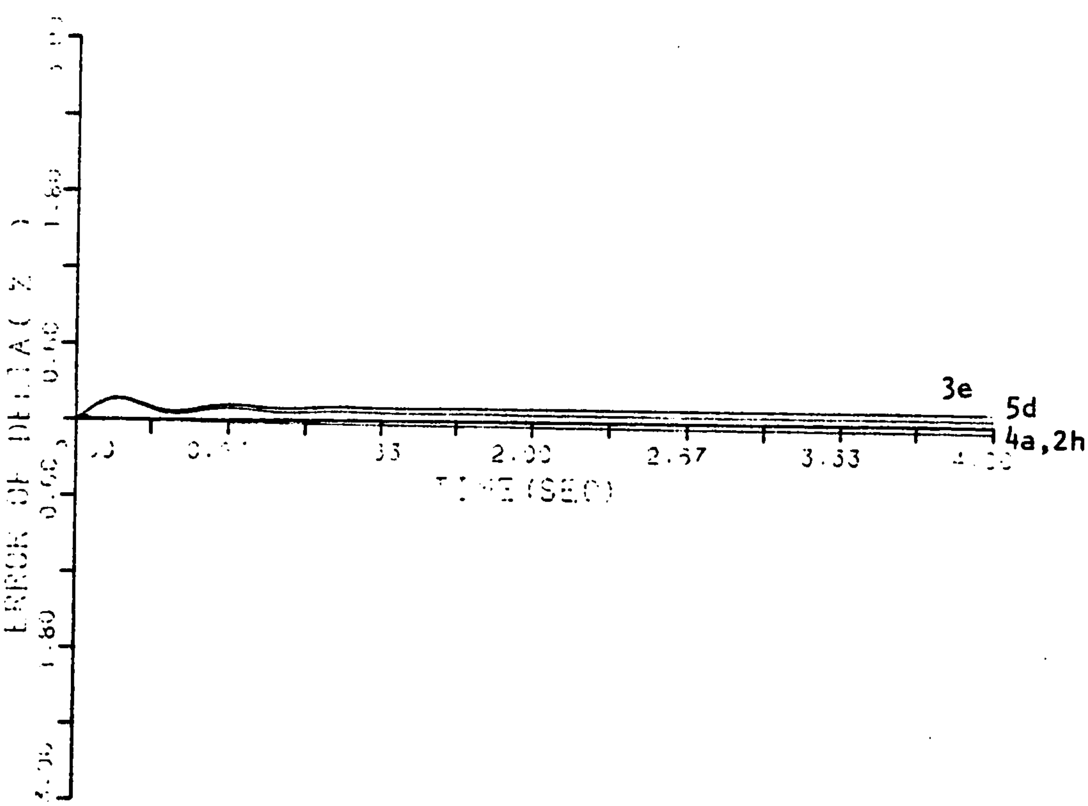


Figure 3.31. Rotor angle error to a 1.0% step change in V_c.

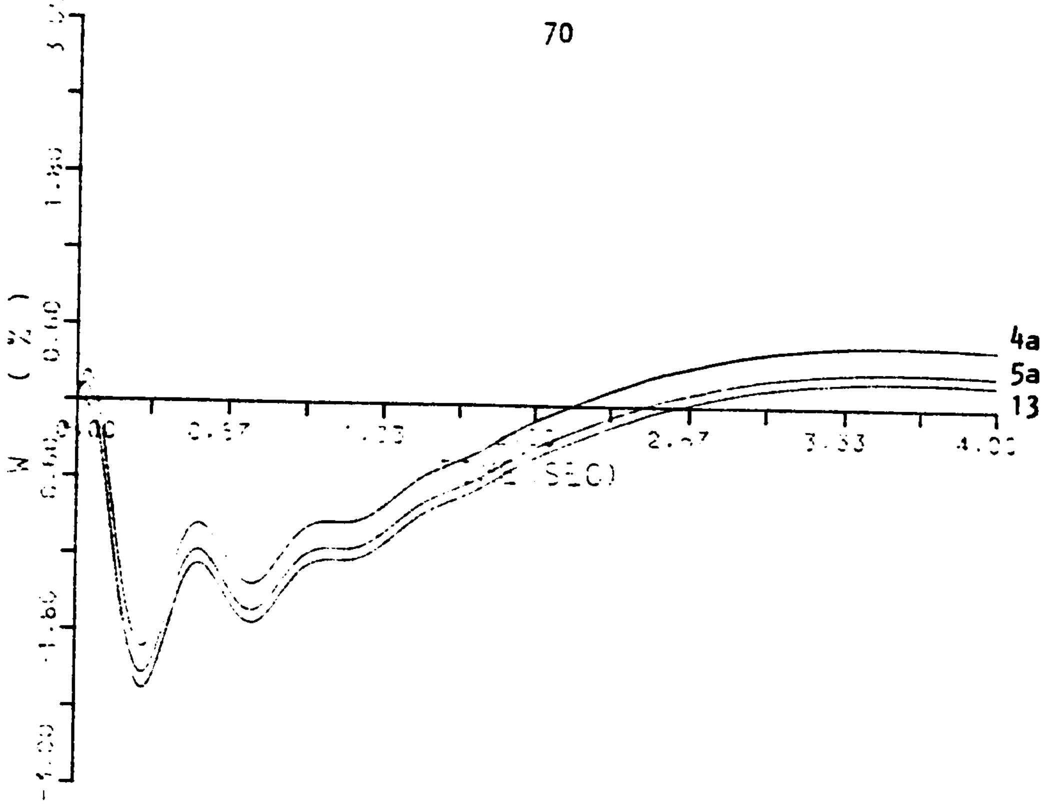


Figure 3.32. Rotor frequency response to a 1.0% step change in V_c.

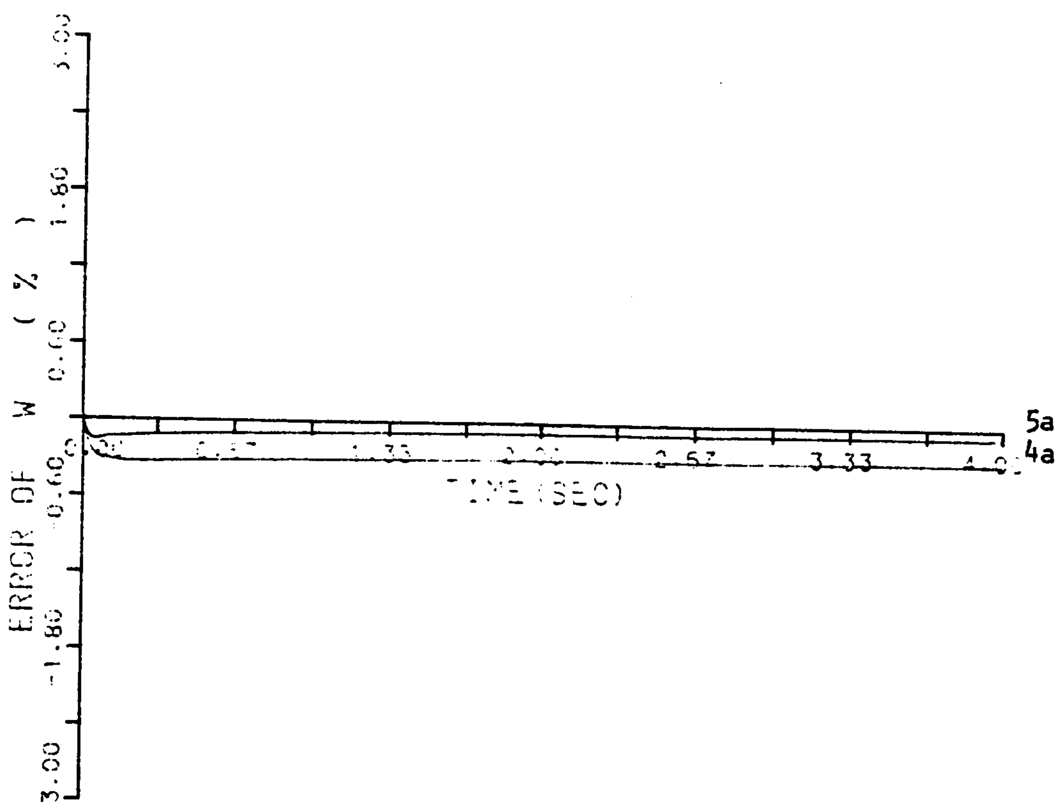


Figure 3.33. Rotor frequency error to a 1.0% step change in V_c.

plotted below each output over the same studied period in Figs. 3.24 through 3.33.

3.7 CONCLUSIONS

In this chapter we utilized a model reduction technique based on the idea explained in chapter 2. We consider the case of multi inputs which is the change in the load power, the change in the setting of the exciter reference voltage and the change in the setting of the governor power one at a time. Four factors are necessary to build a modal reduction technique. These factors are:

1.	Output of interest	y
2.	Input of interest	u
3.	Accuracy	ε
4.	Study time	T

Factors 1 and 2 are associated together because we need to see the effect of the change in certain output corresponding to a change in certain input. As an example the effect of changing the load power on certain output such as the rotor angle.

The input-output performance index is strictly a cause and effect approach. The values of the input time function u represent the cause, and the values of the output y represent the effect.

The accuracy factor depends on the criteria required for a model with permissible error.

Finally, the study time factor is evident from the responses of the reduced model plotted against the complete model, we see that some models are accurate during the initial response while not accurate during the steady state response.

The idea of the input-output performance index is clear from the results obtained and it corrects the way for choosing the important eigenvalues because although some eigenvalues are closer to the imaginary axis, it may not affect the response for some output.

Chapter 4

COMPARISON WITH CLASSICAL ENGINEERING MODEL

4.1 INTRODUCTION

In the previous chapters we have dealt with a mathematical model of the synchronous machine, taking into account the various effects introduced by different rotor ciruits i.e both field effects and damper-winding effects. The model includes five nonlinear differential equations for the synchronous machine windings. In addition to these, other equations describing the governor & turbine system, the shaft, the AVR and the excitation system were considered.

In a stability study the response of a large number of synchronous machines to a given disturbance is investigated, the complete mathematical description of the system would therefore be very complicated unless some simplifications were used. Often only few machines are modeled in details, usually those nearest to the disturbance, while others are described by simpler models. The simplifications depend upon the location of the machine with respect to the disturbance causing the transient and upon the type of disturbance being investigated. As an example, the synchronous machine that consist of two stator circuits and three rotor circuits, a situation may arise in which the dynamics of some of these rotor circuits or their effects can be neglected. Some of the more commonly used simplified models are discussed in this chapter. The underlying assumptions as well as the justifications for their use are briefly outlined. Generally, Table 4.1 shows some of these simplified models and are tabulated in the reverse order of their simplicity.

4.2 COMPENSATED REDUCED ORDER MODELS

Table 4.1 shows some of the classical engineering models often used. These models are based on reproducing only the active components in the generating unit. Actually in these models we ignore some dynamic modes. As an example, for a second order model, the generating unit is represented only by the shaft, which imply that we ignore the governor & turbine system, machine windings and the excitation system. These unrepresented modes include some dynamics and neglecting them affects the accuracy of the model.

Our aim now is to compare the classical engineering models described in Table 4.1 with the corresponding reduced order models described in Chapter 2 (i.e. mathematically reduced order models by using modal analysis). This may be attained by modifying the parameters of the classical models to compensate for the effects of the ignored dynamic parts. The elements of the reduced order models (modal equivalents) matrices are compared with the elements of the matrices of the classical models which are functions in the parameters of the generating unit. Now, the parameters of the classical models are modified in a way such as we add small value to the damping, the inertia, the open circuit time constant etc.

TABLE 4.1. Classical Engineering Models.

Model Order	State Variables δ	ω	E_q'	E_{FD}	E_d'	Comments
2	✓	✓				Ignoring generator windings dynamics i.e. represented by constant voltage behind transient reactance.
3	✓	✓	✓			Adding field dynamic represented by T_{do}'
4	✓	✓	✓	✓		Adding Exciter dynamic represented by one time constant T_E
5	✓	✓	✓	✓	✓	Adding quadrature axis damper winding represented by T_{qo}'

Finally, for similar matrices such as the A matrix, solve for the unknown parameters, so, a new value for the parameters is obtained which leads to force the classical models to satisfy the constraints satisfied by the modal equivalents. The machine was set to the same terminal conditions as in the full system discussed in Chapter 3, with identical V_q, V_d, I_q and I_d.

4.3 CALCULATION OF THE PARAMETERS OF THE SYNCHRONOUS MACHINE

Using the data given in Table 3.1 the parameters of the simplified model are calculated as follow

$$D = 0$$

$$M = 2H/\Omega_B = .014 \text{ sec}^2$$

4.3.1 Synchronous Parameters

$$x_d = x_{md} + x_{la} = 1.56 + .093 = 1.653 \text{ p.u.}$$

$$x_q = x_{mq} + x_{la} = 1.47 + .093 = 1.563 \text{ p.u.}$$

$$x_{fd} = x_{md} + x_{lfd} = 1.56 + .086 = 1.646 \text{ p.u.}$$

$$x_{kq} = x_{mq} + x_{lkq} = 1.47 + .032 = 1.502 \text{ p.u.}$$

$$x_{kd} = x_{md} + x_{lkd} = 1.56 + .048 = 1.608 \text{ p.u.}$$

$$x_e = .2 \text{ p.u.}$$

4.3.2 Transient and Subtransient Parameters

$$x_d' = x_d - (x_{md}^2/x_{fd}) = .1745 \text{ p.u.}$$
$$x_q' = x_q - (x_{mq}^2/x_{kq}) = .124 \text{ p.u.}$$
$$x_d'' = x_d - x_{md}^2(xkd+x_{fd}-2x_{md})/(x_{fd}x_{kd}-x_{md}^2)$$
$$= .1232 \text{ p.u.}$$
$$x_q'' = x_q' = .124 \text{ p.u.}$$
$$T_{do}' = x_{fd}/r_{fd}\Omega_B = 4.37 \text{ sec.}$$
$$T_{do}'' = x_{kd}x_{fd}-x_{md}^2/x_{fd}r_{kd}\Omega_B = .03123 \text{ sec.}$$
$$T_d'' = T_{do}''\, x_d''/x_d' = .022 \text{ sec.}$$
$$T_d' = T_{do}' x_d'/x_d = .4613 \text{ sec.}$$
$$T_{qo}' = x_{kd}/r_{kd}\Omega_B = 0.285 \text{ sec.}$$
$$T_{qo}'' = T_{qo}'$$
$$T_q'' = T_{qo}''/x_q = .0226 \text{ sec}$$
$$T_a = (x_d'+x_q)/2r_a\Omega_B = .72012 \text{ sec.}$$

4.4 CLASSICAL SECOND ORDER ENGINEERING MODEL

If we assume that the voltage E_q' which corresponds to the d axis field flux linkage, changes at a rate that depends upon T_{do}'. This time constant is on the order of several seconds. The voltage E_{FD} (output voltage of the exciter) depends on the excitation system characteristics. If E_{FD} does not change very fast and if the impact initiating the transient is short, so in some cases the assumption that the voltage E_q' remains constant during the transient response can be justified. Another assumption is to ignore the saliency effect, which implies that $E'_q = E'_d = E'$. According to these assumptions the equations describing the rotor and the stator become algebraic equations. Now, the only dynamic part in the generating unit is the shaft, and it is represented as second order. In this model all the dynamics of the machine windings, the governor/turbine system and the excitation system are ignored.

4.4.1 Machine Windings Representation

The machine windings are represented by an algebraic equation which is a voltage behind a transient reactance

$$E' = V_t + j X'_d I \tag{4.1}$$

4.4.2 Shaft System Representation

The differential equations of this model are derived in detail in Appendix A4-I which are

$$\dot{\Delta\delta} = \Delta\omega \tag{4.2}$$

$$\dot{\Delta\omega} = -\frac{K}{M}\Delta\delta - \frac{D}{M}\Delta\omega + \frac{1}{M}\Delta P_L \tag{4.3}$$

where

$$K = |V_B E'| \cos \delta_o / X$$

$$X = X'_d + X_e$$

4.4.3 Complete Classical Second Order Engineering Model for Generating Unit

Equations 4.2 and 4.3 represent a classical second order engineering model for a complete generating unit. The state variables are

$$X_2^t = [\Delta\delta \quad \Delta\omega]$$

The equations describe a classical second order engineering model are summarized in Table 4.2.

The state variables form in this model can be written as

$$\dot{X}_2 = A_2 X_2 + B_2 u \qquad (4.4)$$

where

$$A_2 = \begin{array}{c} \begin{array}{cc} \Delta\delta & \Delta\omega \end{array} \\ \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & 0 \end{bmatrix} \end{array} \quad \text{and}$$

$$B_2 = \begin{bmatrix} 0 \\ +\frac{1}{M} \end{bmatrix}$$

TABLE 4.2. Summary of Equations for a Classical Second Order Engineering Model.

Differential Equations:

$$\dot{\Delta\delta} = \Delta\omega \tag{4.2}$$

$$\dot{\Delta\omega} = -\frac{K}{M}\Delta\delta - \frac{D}{M}\Delta\omega + \frac{1}{M}\Delta P_L \tag{4.3}$$

Algebraic Equation:

$$E' = V_t + j X'_d I \tag{4.1}$$

Units:

t in second, ω in rad/sec, M in p.u. power sec^2, D in p.u. power sec. currents voltage and power in p.u.

38b

The constant K can be calculated as follow

$$K = |V_B E'| \cos \delta_o / X$$

$$E' = V_B + j\,XI$$

$$X = X_e + X_e'$$

Substituting from Table 4.2 we get

$$V_t = 1.131 \underline{/8.1}^o$$

and

$$E' = V_B + j\,XI$$

$$= 1.2606 \underline{/13.7}^o$$

and $K = 3.273133$

The numerical value for A_2 is

$$\begin{array}{cc} & \begin{array}{cc} \Delta\delta & \Delta\omega \end{array} \\ A_2 = & \begin{pmatrix} 0 & 1.0 \\ -233.79 & 0 \end{pmatrix} \end{array}$$

The eigenvalues of this model are

$$\lambda_{1,2} = \pm\, j15.114$$

4.5 COMPENSATED CLASSICAL SECOND ORDER

The state equation of the second order modal equivalent using the mathematical method discussed in Chapter 2 is

$$\dot{X}_2^{*} = A_2^{*} X_2^{*} + B_2^{*} u \tag{4.5}$$

Now, the proposed idea explained in section 4.2 can be applied to the model of Eqn. (4.5) and the model of Eqn. (4.4) by comparing the elements of the matrices A_2 and A_2^{*}. Also for other similar matrices such as B_2 & B_2^{*}, C_2 & C_2^{*} and D_2 & D_2^{*}.

The elements of the matrices A_2 and A_2^{*} are defined as follows:

$$A_2^{*} = \begin{array}{c} \begin{array}{cc} \Delta\delta & \Delta\omega \end{array} \\ \begin{pmatrix} A_{2,11}^{*} & A_{2,12}^{*} \\ A_{2,21}^{*} & A_{2,22}^{*} \end{pmatrix} \end{array} \tag{4.6}$$

$$A_2 = \begin{array}{c} \begin{array}{cc} \Delta\delta & \Delta\omega \end{array} \\ \begin{pmatrix} A_{2,11} & A_{2,12} \\ A_{2,21} & A_{2,22} \end{pmatrix} \end{array} = \begin{pmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{D}{M} \end{pmatrix} \tag{4.7}$$

The parameters of 4.7 are modified in a way such as

$$A_2 = \begin{bmatrix} A_{2,11} & A_{2,12} \\ & \\ A_{2,21} & A_{2,22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ & \\ -\dfrac{K}{M+\Delta M} & -\dfrac{D+\Delta D}{M+\Delta M} \end{bmatrix} \tag{4.8}$$

Equating the elements of 4.6 and 4.7 or generally we can write

$$A^*_{2_{ij}} = A_{2_{ij}} \tag{4.9}$$

$$i, j = 1, 2$$

From Chapter 2

$$A^*_{2_{ij}} = M^*_{i\alpha}\ \lambda^*_{\alpha}\ N^*_{\alpha j} \tag{4.10}$$

where M^*, Λ^* and N^* are defined in Chapter 2 or

$$A^*_{2_{11}} = A_{2_{11}} \tag{4.11}$$

$$A^*_{2_{12}} = A_{2_{12}} \tag{4.12}$$

$$A^*_{2_{21}} = A_{2_{21}} \tag{4.13}$$

and

$$A^*_{2_{22}} = A_{2_{22}} \qquad (4.14)$$

The numerical values for 4.6 are

$$\begin{array}{cc} \Delta\delta & \Delta\omega \\ \end{array}$$

$$A^*_2 = \begin{pmatrix} 0 & 1 \\ & \\ -172.613 & -5.301 \end{pmatrix} \qquad (4.15)$$

From 4.7 and 4.13, Equations (4.11) and (4.12) are satisfied, this is a special case when the rotor angle and the rotor speed are considered as state variables.

Then for 4.13 and 4.14

$$-\frac{K}{M + \Delta M} = A^*_{2_{21}} \qquad (4.16)$$

$$-\frac{D + \Delta D}{M + \Delta M} = A^*_{2_{22}} \qquad (4.17)$$

Solve for ΔM and ΔD we get

$$\Delta M = -\frac{K}{A^*_{2_{21}}} - M \qquad (4.18)$$

$$\Delta D = -A^{*}_{2_{22}}(M + \Delta M) = D \qquad (4.19)$$

Substituting we get

$\Delta M = .00496226$

$\Delta D = .100518$

and

$M^{new} = 0.014 + 0.004885 = .0189622$

$D^{new} = .0 + .0980889 = .100518$

The result obtained can be explained by the following:

a- That using a governor to adjust the speed is equivalent to inertia being large, but in the classical second order engineering model the governor and turbine system are neglected. To compensate for these, the inertia should be increased, the result shows that the inertia increased one third of the old value approximately.

b- The damping of a Generating unit is the sum of the damping of the turbine, the machine windings, the transmission network and the damping due to the load or.

$$D = D_T + D_m + D_N + D_L \qquad (4.20)$$

Since we ignore all these parts in our study, so it is necessary to consider some value for the damping. The result shows that the new value for the damping is infinite corresponding to the old value which is equal to zero. This result is acceptable because all sources of damping in this model are neglected especially the damper winding in the q-axis which plays an important part in the damping of the synchronous machine represented by the 13th order model.

According to above, the classical second order engineering model described by 4.4 is compensated to satisfy all the constraints satisfied by the second order model obtained in Chapter 3 using the modal analysis. The output of the reduced (a) second order classical model, (b) second order reduced model, and (c) the compensated classical are calculated and plotted versus the complete response of 13th order model to a 1% step change in the load power, also the error of these models are calculated w.r.t. 13th order model and plotted below each output over the same studied period in Figs. 4.1 through 4.4.

Table 4.3 summarizes the results of the reduced (a), the classical (b) and the compensated classical (c) order models for second order models showing for each model the eigenvalues, the p.u. R.M.S. error and the compensated value for the parameters.

TABLE 4.3. Summary of the Results Obtained for Second Order Model.

Model		Reduced(1)	Classical	Compensated(2) Classical
Eigenvalues	$\lambda_{1,2}$	$-2.574 \pm j12.854$	$\pm j15.114$	$-2.574 \pm j12.854$
p.u.(3) R.M.S. ERROR	δ	.3491	.637	.3491
	ω	1.04	3.183	1.04
	I_d	----	----	----
	E_{fd}	----	----	----
	I_q	----	----	----

(1) Reduced from 13th order model

(2) $\Delta M = .00496226$

$\Delta D = .100518$

(3) p.u. R.M.S. Error $= [\sum_{t_1}^{t_2} (x - x^*)^2/x^2]^{1/2}$

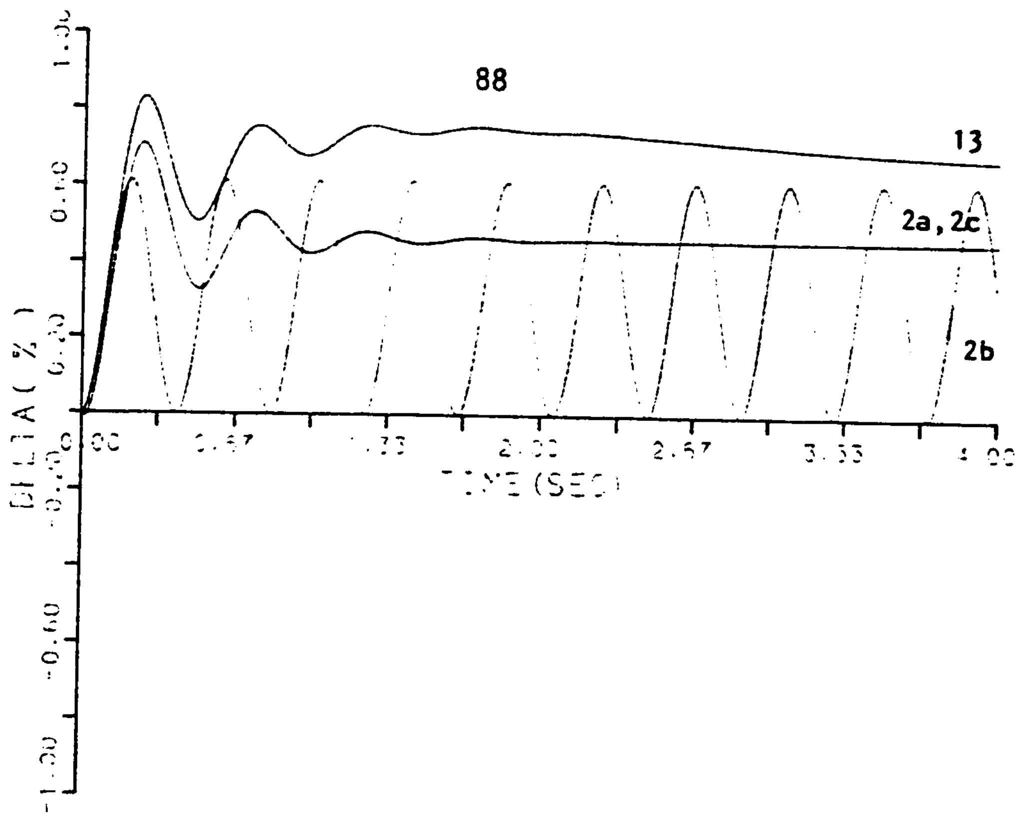


Figure 4.1. Rotor angle response to a 1.0% step change in P_L.

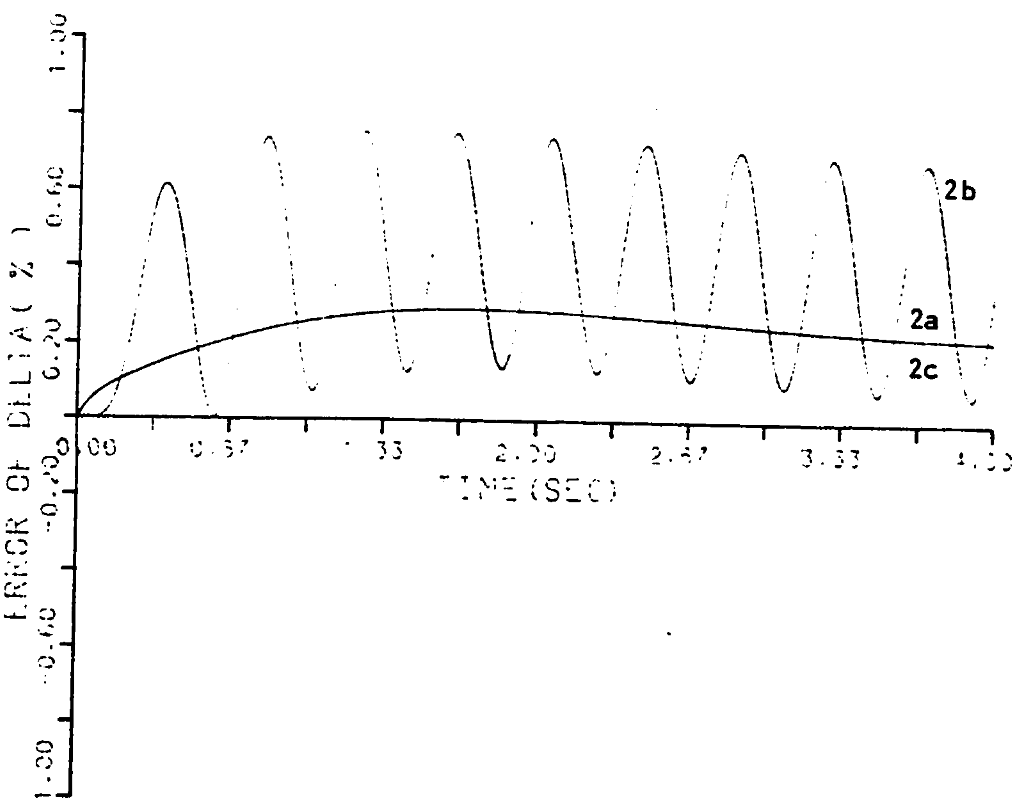


Figure 4.2. Rotor angle error to a 1.0% step change in P_L.

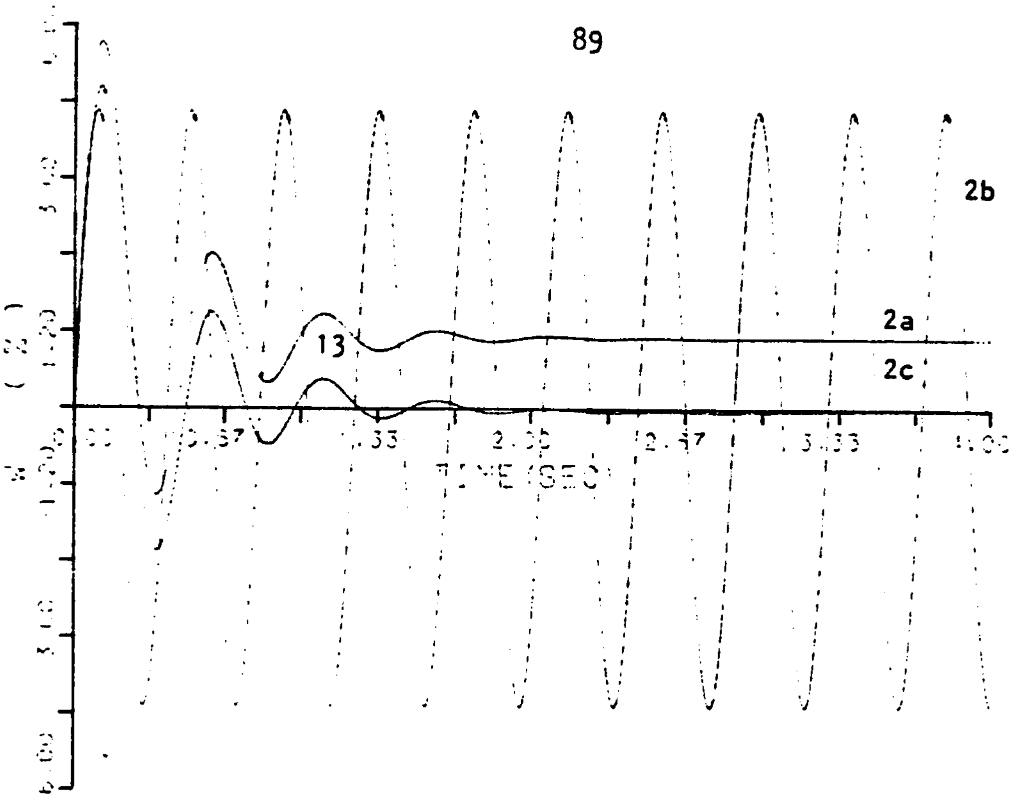


Figure 4.3. Rotor frequency response to a 1.0% step change in P_L.

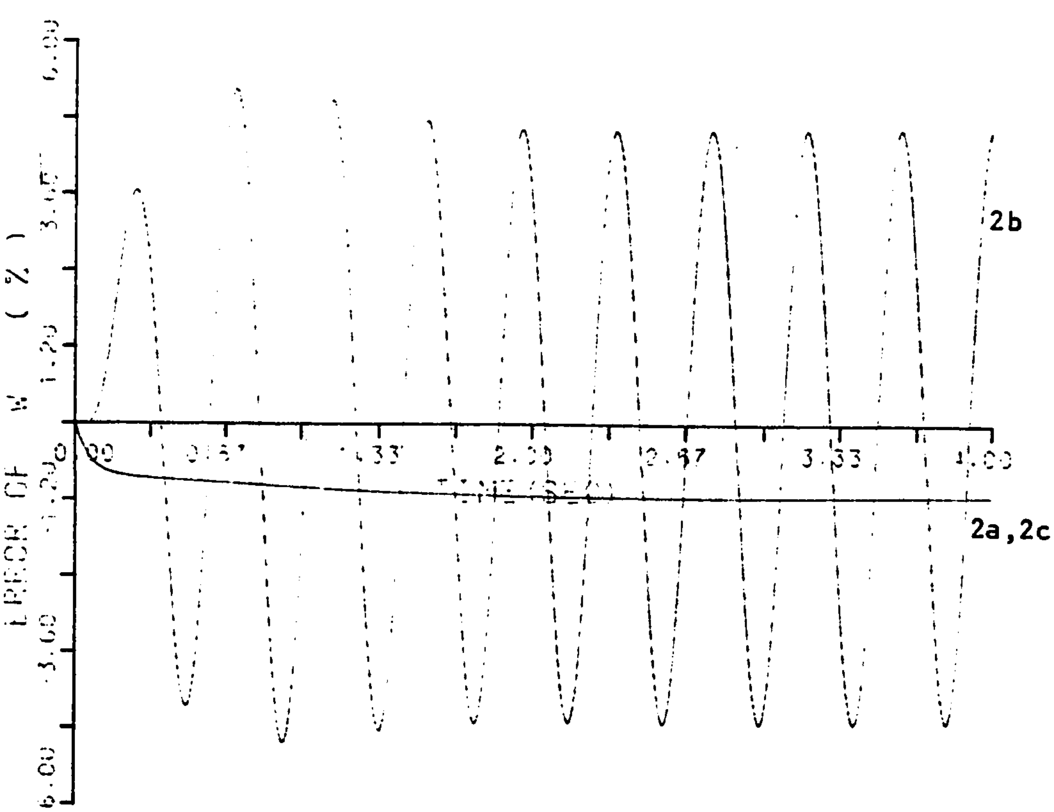


Figure 4.4. Rotor frequency error to a 1.0% step change in P_L.

4.6 CALCULATION OF THE DAMPING POWER FOR THE DAMPER WINDINGS

Kimbark [12] calculates the damping power of a synchronous machine connected to an infinite bus through series reactance assuming the following assumptions:

(1) No resistance in the armature circuit.

(2) No resistance in the field circuit.

(3) Damping action caused only by the damper windings.

$$D = v_{d0}^{2}(x'_d - x''_d)\, T'_{d0}/(x'_d + x_e)^2 + v_{q0}^{2}(x_q - x'_q)\, T'_{q0}/(x_q + x_e)^2$$

$$= D_d + D_q$$

Where

- D instantaneous damping power in p.u. power sec.
- D_d direct axis damping power.
- D_q quadrature axis damping power.

Using the parameters of the synchronous machine calculated in section 4.3, the d-axis damping power and the q-axis damping power can be calculated as follows:

$$\text{Direct axis damping power} = (.564)^2(.1745-.1232)(.03123)/(.2+.1745)^2$$

$$= .00363365 \text{ p.u. power sec.}$$

Quadr. axis damping power = $(.825)^2(1.563-.124)(.285)/(.2+1.563)^2$

= .08906738 p.u. power sec.

D = .00363765 + .08906738 = .092701 p.u. power sec.

This value is very close to the value obtained by compensation ΔD = .1005, this value D can be used in the second, third and fourth classical models to compensate for ignoring the damper windings in these models. The result shows that the D_q is larger than D_d which means that the positive damping in the synchronous machine is provided from the q-axis damper winding. In the next sections we shall assume that D due to the damper windings is known and equal to .092701 p.u. power sec. The classical third and fourth order engineering models are now modified to incorporate the effects of the damper windings which mainly provided from the q-axis.

4.7 CLASSICAL THIRD ORDER ENGINEERING MODEL

In this model the transient effects of one of the rotor circuit is represented, which is the field circuit in the direct axis. Also saliency effect is considered. These assumptions yield a first order for the machine windings. Adding this to a second order for the shaft yields a third order model for a complete generating unit. Again in this model the governor, turbine and excitation system dynamics are neglected.

4.7.1 Machine Windings Representation

As mentioned above, the machine windings are represented by a first order. The differential equation describing this model is derived in detail in Appendix A4-II which is

$$\Delta \dot{E}'_q = -\frac{\Delta E'_q}{T'_{do}} - \frac{(X_d - X'_d)}{T'_{do}}\Delta i_d + \frac{\Delta E_{FD}}{T'_{do}} \tag{4.23}$$

Also the algebraic equations describing the stator voltage components neglecting stator resistance are:

$$\Delta V_q = -X'_d\,\Delta i_d + \Delta E'_q \tag{4.24}$$

$$\Delta V_d = X_q\,\Delta i_q \tag{4.25}$$

Figure 4.5 shows a phasor diagram of asynchronous machine during transient explaining equations (4.24) and (4.25).

From Fig. 4.5 the stator voltage components can be expressed in terms of the rotor angle

$$V_q = V_B \cos \delta_o \tag{4.26}$$

$$V_d = V_B \sin \delta_o \tag{4.27}$$

Equations (4.26) and (4.27) for small perturbation are

$$\Delta V_q = -V_B \sin \delta_o \, \Delta\delta \qquad (4.28)$$

$$\Delta V_d = V_B \cos \delta_o \, \Delta\delta \qquad (4.29)$$

Substituting (4.28) and (4.29) into (4.24) and (4.25) yields

$$\Delta E_q' = -V_B \sin \delta_o \, \Delta\delta + X_d' \, \Delta i_d \qquad (4.30)$$

$$0 = -V_B \cos \delta_o \, \Delta\delta + X_q \, \Delta i_q \qquad (4.31)$$

4.7.2 Machine Windings Representation Including the Infinite Bus and Transmission Line

The direct and quadrature axis reactances modified by adding the transmission line reactances as an example

$$\hat{X}_d = X_d + X_e \qquad (4.32)$$

$$\hat{X}_q = X_q + X_e \qquad (4.31)$$

and by the same way the transient and subtransient reactances can be modified. Equations (4.30) and (4.31) become

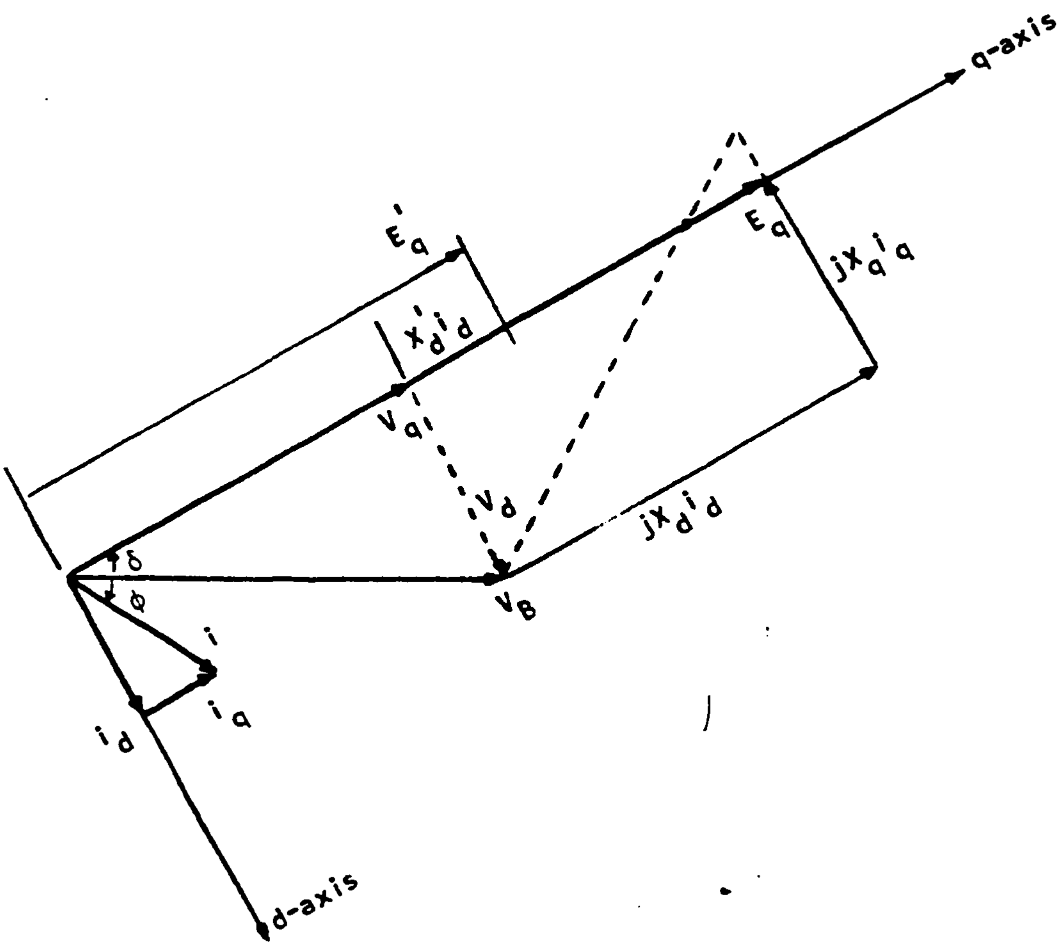


Figure 4.5. Phasor diagram of a synchronous machine during transient.

$$\Delta E_q' = -V_B \sin \delta_o \, \Delta\delta + (X_d' + X_e)\, \Delta i_d \tag{4.34}$$

$$0 = -V_B \cos \delta_o \, \Delta\delta + (X_q + X_e)\, \Delta i_q \tag{4.35}$$

Since we are interested in the direct axis current as a state variable, it is necessary to change Eqn. (4.23) to incorporate the direct axis current as a state variable instead of E_q'.

Substituting (4.34) in (4.23) we get

$$(X_d' + X_e)\, \Delta \dot{i}_d - V_B \, \delta_o \sin \Delta\dot{\delta} = -\frac{X_d' + X_e}{T_{do}'} \Delta i_d + \frac{V_B \sin \delta_o}{T_{do}'} \Delta\delta$$

$$+ \frac{X_d' - X_d}{T_{do}'} \Delta i_d + \frac{\Delta E_{FD}}{T_{do}'} \tag{4.36}$$

rearranging Eqn. (4.36) considering $\Delta E_{FD} = 0$ we get

$$(X_d' + X_e)\, \Delta \dot{i}_d - V_B \sin \delta_o \, \Delta\dot{\delta} = -\frac{X_d + X_e}{T_{do}'} \Delta i_d + \frac{V_B \sin \delta_o}{T_{do}'} \Delta\delta \tag{4.37}$$

4.7.3 Shaft System Representation

The shaft is represented by a second order model neglecting the turbine and governor system dynamics. The state variables are the rotor angle and the rotor frequency. The differential equations describe this model are

$$\dot{\Delta\delta} = \Delta\omega \tag{4.38}$$

$$\dot{\Delta\omega} = \frac{\Delta P_m}{M} - \frac{D}{M}\Delta\omega - \frac{\Delta P_e}{M} + \frac{\Delta P_L}{M} \tag{4.39}$$

Assuming that $\Delta P_m = 0$ (i.e. the mechanical power is constant). The electrical power ΔP_e can be obtained from

$$P_e = V_q i_q + V_d i_d \tag{4.40}$$

Substituting (4.28) and (4.29) into (4.40)

$$P_e = V_B \cos\delta\, i_q + V_B \sin\delta i_d \tag{4.41}$$

$$\Delta P_e = V_B \cos\delta_o \Delta i_q - V_B I_{qo} \sin\delta_o \Delta\delta + V_B \sin\delta_o \Delta i_d + V_B I_{do} \cos\delta_o \Delta\delta \tag{4.42}$$

Substituting (4.35) in (4.42)

$$\Delta P_e = (V_B \sin\delta_o)\Delta i_d + \left(V_B I_{do} \cos\delta_o - V_B I_{qo} \sin\delta_o + \frac{(V_B \cos\delta_o)^2}{X_q + X_e}\right)\Delta\delta \tag{4.43}$$

or

$$\Delta P_e = V_{do} \Delta i_d + \left(V_{qo} I_{do} - V_{do} I_{qo} + \frac{V_{qo}^2}{X_q + X_e}\right)\Delta\delta \tag{4.44}$$

$$\Delta P_e = K_{31}\,\Delta i_d + K_{32}\Delta\delta \qquad (4.45)$$

where

$$K_{31} = V_{do}$$

and

$$K_{32} = \left(V_{qo}\, I_{do} - V_{do}\, I_{qo} + \frac{V_{qo}^2}{X_q + X_e}\right)$$

Substituting (4.45) in (4.39)

$$\dot{\Delta\omega} = -\frac{D}{M}\,\Delta\omega - \frac{K_{31}}{M}\,\Delta i_d - \frac{K_{32}}{M}\,\Delta\delta + \frac{1}{M}\Delta P_L \qquad (4.46)$$

4.7.4 Complete Generating Unit Model

Equations (4.37), (4.38) and (4.46) represent a simplified 3rd order model for a complete generating unit. The state variables are

$$X_3^t = [\Delta i_d\ \Delta\delta\ \Delta\omega]$$

The equations describing a classical third order engineering model are summarized in Table 4.4.

TABLE 4.4. Summary of Equations for a Classical Third Order Engineering Model.

<u>Differential Equations</u>

$$(X_d' + X_e)\,\Delta\dot{i}_d - V_B \sin\delta_o\,\Delta\dot{\delta} = -\frac{X_d + X_e}{T_{do}'}\,\Delta i_d + \frac{V_B \sin\delta_o}{T_{do}'}\,\Delta\delta \qquad (4.37)$$

$$\Delta\dot{\delta} = \Delta\omega \qquad (4.38)$$

$$\Delta\dot{\omega} = -\frac{D}{M}\,\Delta\omega - \frac{K_{31}}{M}\,\Delta i_d - \frac{K_{32}}{M}\,\Delta\delta + \frac{1}{M}\,\Delta P_L \qquad (4.46)$$

<u>Algebraic Equations</u>

$$\Delta P_e = K_{31}\Delta i_d + K_{32}\Delta\delta \qquad (4.45)$$

where $K_{31} = V_{do}$

$$K_{32} = \left(V_{qo}\, I_{do} - V_{do}\, I_{qo} + \frac{V_{qo}^2}{X_q + X_e}\right)$$

<u>Units:</u> t in second, ω in rad/sec, M in p.u. power sec ^{2}, D in p.u. power sec, currents, voltage and power in per unit.

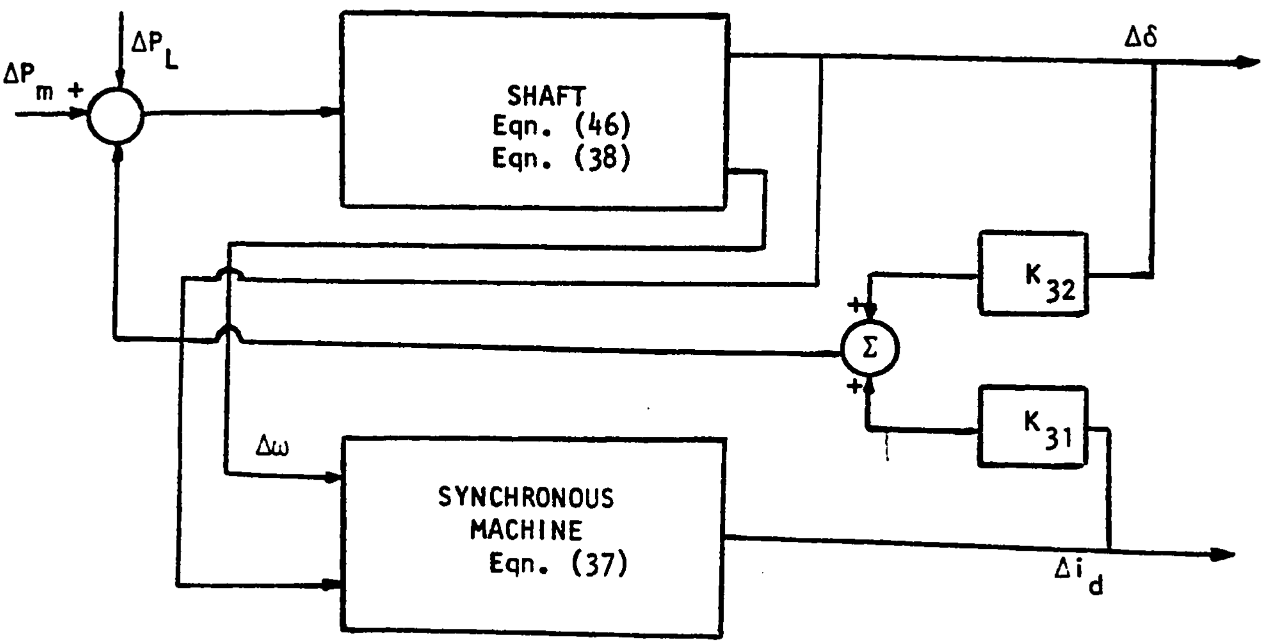


Figure 4.6. One machine infinite bus system dynamic subassemblies parameters for a classical third order Engineering model.

Figure 4.6 shows a one machine infinite bus system dynamic subassemblies parameters for a classical third order Engineering model.

The state variables form for this model can be written as

$$L_e \dot{X}_3 = Z_3 X_3 + B_3 u \tag{4.47}$$

where

$$L_3 = \begin{array}{c} \begin{array}{ccc} \Delta i_d & \Delta\delta & \Delta\omega \end{array} \\ \begin{pmatrix} X_d' + X_e & -V_B \sin\delta_0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

$$Z_3 = \begin{array}{c} \begin{array}{ccc} \Delta i_d & \Delta\delta & \Delta\omega \end{array} \\ \begin{pmatrix} -\dfrac{X_d + X_e}{T_{d0}'} & \dfrac{V_B \sin\delta_0}{T_{d0}'} & 0 \\ 0 & 0 & 1 \\ -\dfrac{K_{31}}{M} & -\dfrac{K_{32}}{M} & -\dfrac{D}{M} \end{pmatrix} \end{array}$$

Inverting the matrix L_3 and substituting in Eqn. (4.47)

$$\dot{X}_3 = A_3 X_3 + L_3^{-1} B_3 u \tag{4.48}$$

$$\dot{X} = A_3 X_3 + B_3 u \tag{4.49}$$

where

$$A_3 = \begin{array}{c} \begin{array}{ccc} \Delta i_d & \Delta\delta & \Delta\omega \end{array} \\ \begin{bmatrix} -\dfrac{K_{11}}{T'_{d0}} & \dfrac{K_{12}}{T'_{d0}} & V_B \operatorname{Sin} \delta_0 \\ 0 & 0 & 1 \\ -\dfrac{K_{31}}{M} & -\dfrac{K_{32}}{M} & -\dfrac{D}{M} \end{bmatrix} \end{array}$$

and

$$K_{11} = \frac{X_d + X_e}{X'_d + X_e}$$

$$K_{12} = \frac{V_B \sin \delta_o}{X'_d + X_e}$$

$$K_{31} = V_{do}$$

$$K_{32} = \left(V_{qo}\, I_{do} - V_{do}\, I_{qo} + \frac{V_{qo}^2}{X_q + X_e}\right)$$

$$B_3 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \end{bmatrix}$$

Using the data and the operating point calculated in section 4.3, the numerical value for the K's can be calculated as follows.

$K_{11} = 4.954$ p.u.

$K_{12} = 1.506754$ p.u.

$K_{31} = .564$ p.u.

$K_{32} = .9849$ p.u.

$D = .092701$ p.u. power sec.

Noting that the Damping D is used to be equal to the Damping calculated using Kimbark formula as stated at the end of section 4.6.

Using the values of the K's calculated above and using the typical parameters of the generating unit the matrix A_3 can be calculated as follows.

$$\begin{array}{c} \begin{array}{ccc} \Delta i_d & \Delta\delta & \Delta\omega \end{array} \\ A_3 = \begin{bmatrix} -1.13364 & .34479 & 1.50675 \\ 0 & 0 & 1 \\ -40.28 & -70.3502 & -6.6215 \end{bmatrix} \end{array}$$

The eigenvalues of this model are

$$\lambda_{1,2} = -\,2.634 \pm j12.786$$
$$\lambda_3 = -\,.634$$

However, the third order reduced model has the eigenvalues

$$\lambda^*_{1,2} = 2.634 \pm j12.787$$
$$\lambda^*_3 = -\,.636$$

which is by inspection very close to the classical third order Engineering model.

Noting that A^*_3 is obtained from reducing the 7th order model (2 for the shaft and 5 for synchronous machine) to third order model and not from the 13th order model as normally used for the comparison in this present chapter. The reason for that is at the 13th order model we cannot get a reduced 3rd order model because in the 13th order model the field circuit is coupled with the direct axis producing a complex conjugate of eigenvalues. This implies that the direct axis current and the exciter output voltage must be represented in the reduced model which would be a 4th order when these 2 state variables are added to the δ and ω state variables.

Table 4.5 summarizes the results of the reduced order model and the classical third order Engineering model showing the eigenvalues, and the p.u. R.M.S. error.

TABLE 4.5. Summary of the Results Obtained for Third Order Model.

	Model	Reduced(1)	Classical
Eigenvalues	$\lambda_{1,2}$	$-2.634 \pm j12.787$	$-2.634 \pm j12.786$
	λ_3	$-.636$	$-.634$
p.u.(2) R.M.S. ERROR	δ	.077	.048
	ω	.87	.348
	I_d	.221	.0973
	E_{fd}	----	----
	I_q	----	----

(1) Reduced from 7th order model

(2) p.u. R.M.S. Error = $[\sum_{t_1}^{t_2} (x - x^*)^2/x^2]^{1/2}$

360

As a conclusion, the results obtained show that accuracy of the classical third order Engineering model is better than the reduced third order model. According to this, it is not needed to compensate for the parameters of the synchronous machine.

The output of the reduced third order model, (a) and the classical (b) third order Engineering model are calculated and plotted versus the complete response of 7th order model to a 1% step change in the load power, also the error of these models are calculated w.r.t. 7th order model and plotted down each output over the same studied period in Figs. 4.7 through 4.12.

4.8 CLASSICAL FOURTH ORDER ENGINEERING MODEL

In this model the coupling between the exciter and the direct axis is considered. The transient effects are dominated by the field circuit in the direct axis and the excitation system. According to these assumptions the machine windings are represented only by a first order which is the quadrature axis voltage E_q' caused by the flux linkage of the direct axis. The excitation system is represented by a first order model by considering the exciter output voltage as a state variable. The change in the voltage E_{FD} is related to a change in the terminal voltage V_t of the synchronous machine. Adding these to a second order model for the shaft yields a fourth order model for a complete generating unit. Again the dynamics of the turbine and the damper windings are neglected in this model but the damper windings effects are

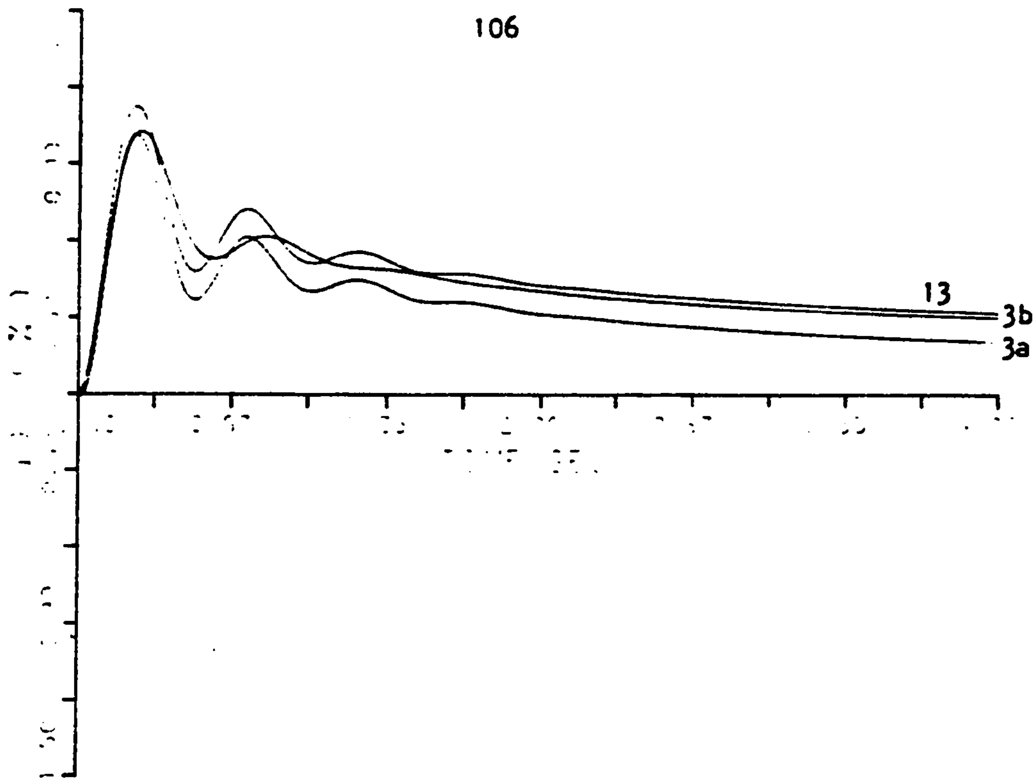


Figure 4.7. I_d response to a 1.0% step change in P_L.

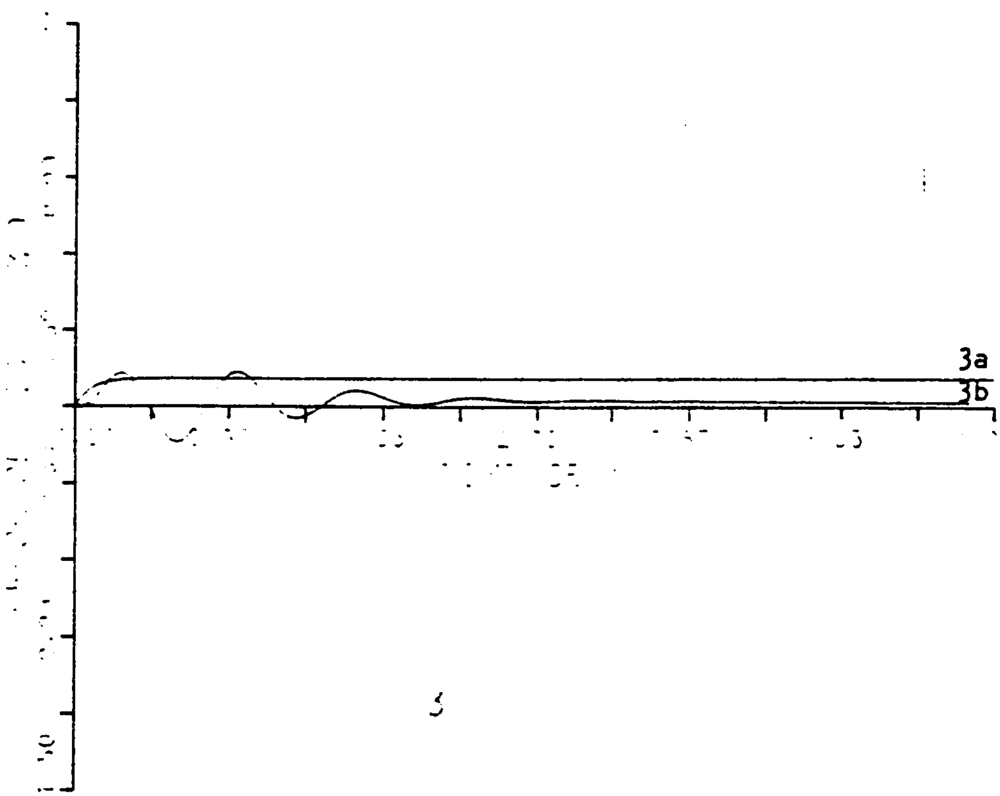


Figure 4.8. I_d error to a 1.0% step change in P_L.

360

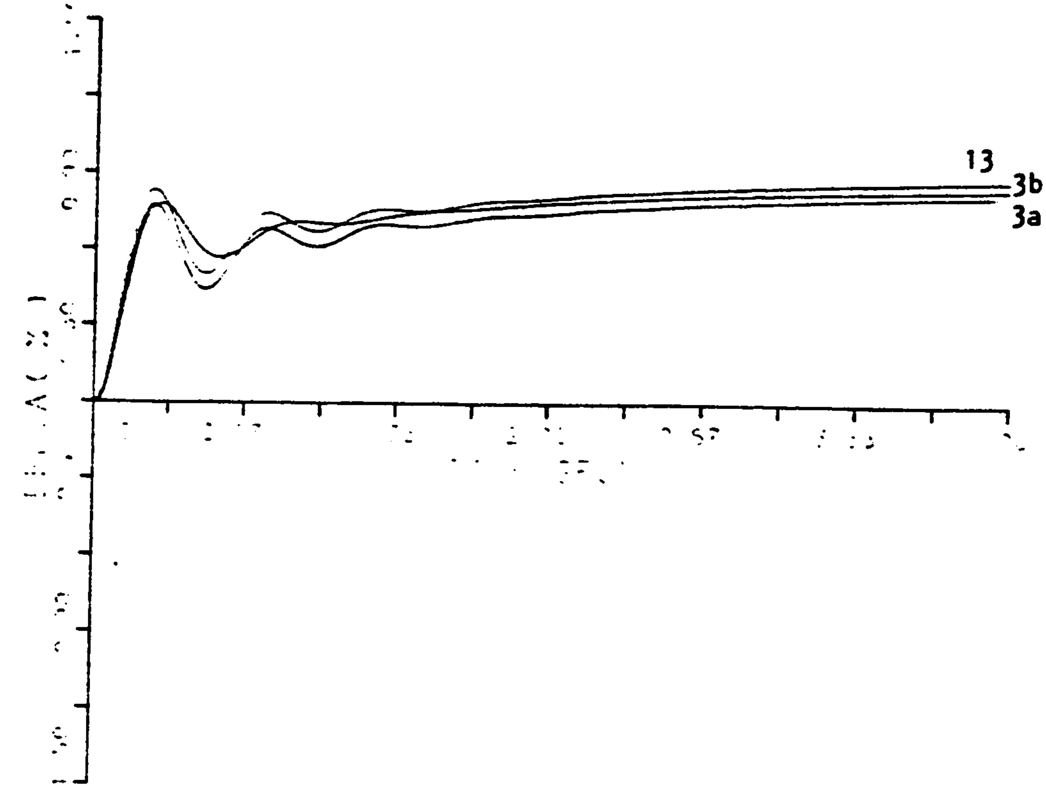


Figure 4.9. Rotor angle response to a 1.0% step change in P_L.

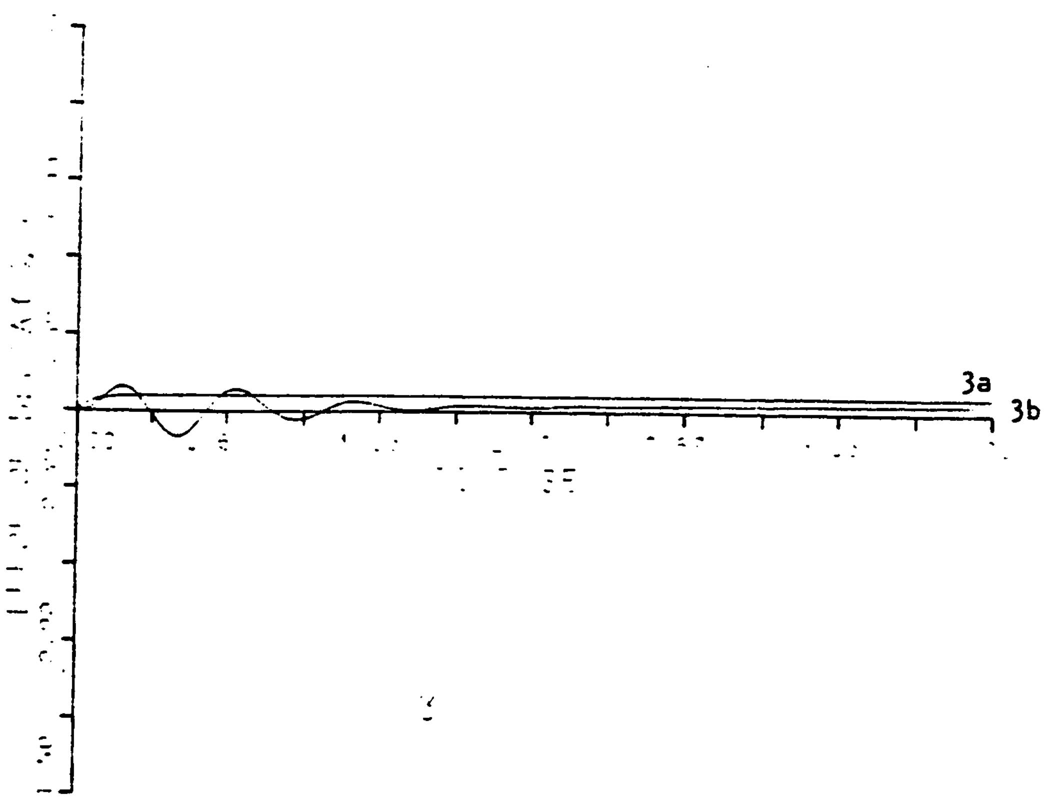


Figure 4.10. Rotor angle error to a 1.0% step change in P_L.

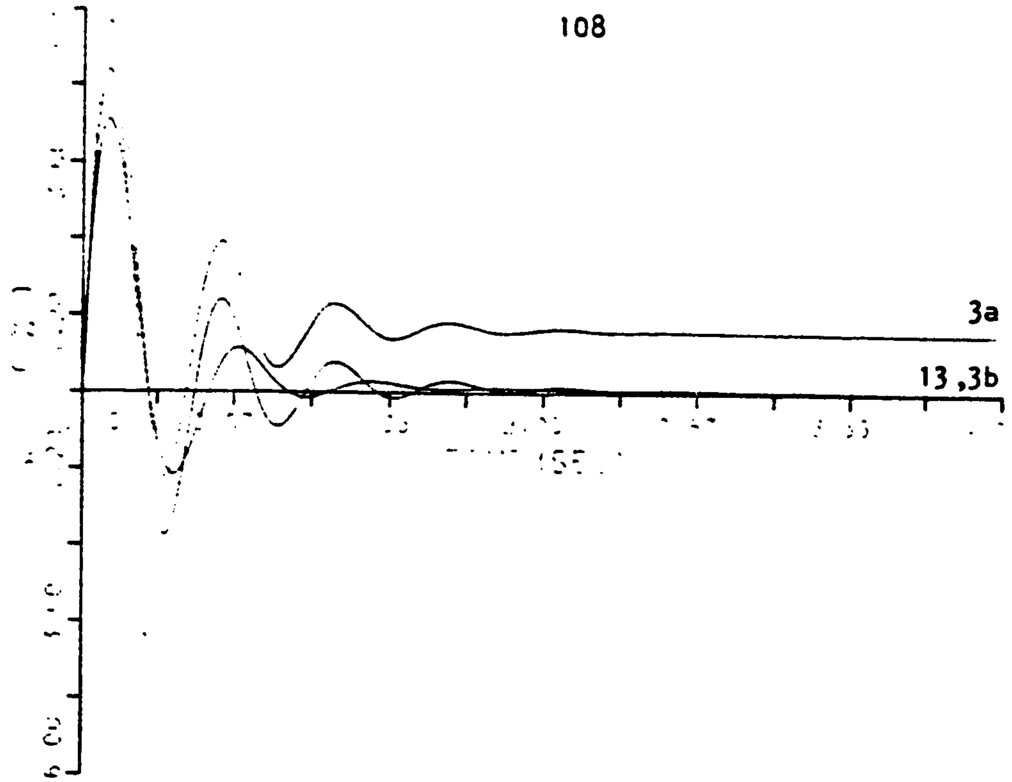


Figure 4.11. Rotor frequency response to a 1.0% step change in P_L.

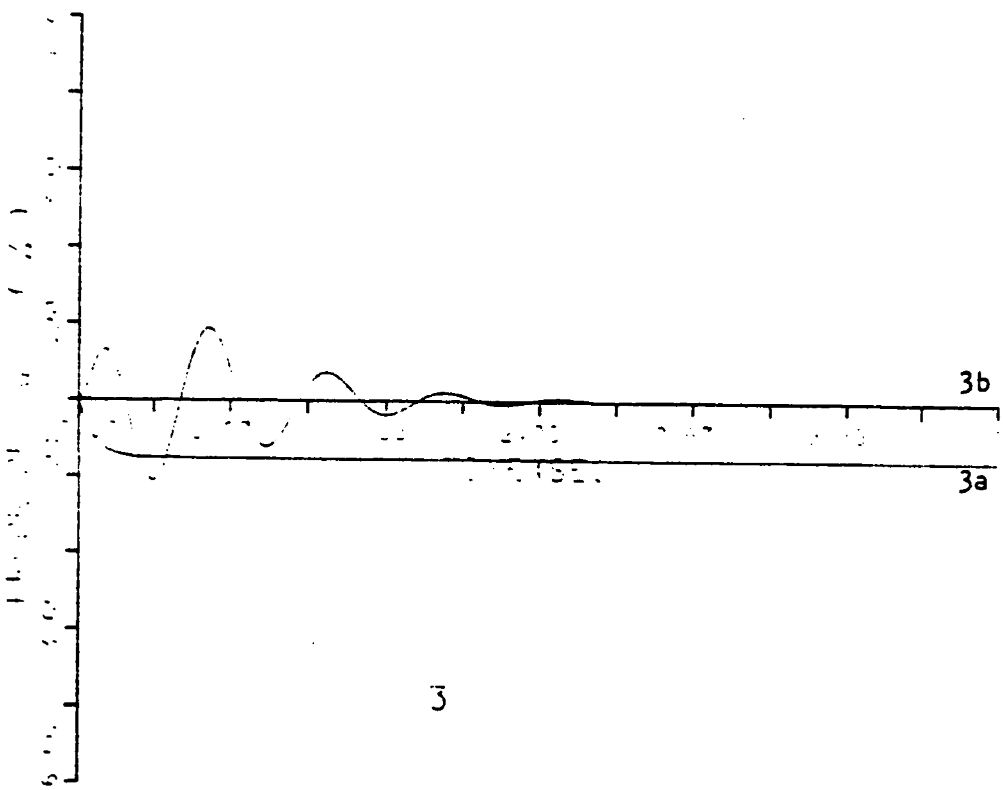


Figure 4.12. Rotor frequency error to a 1.0% step change in P_L.

360

represented by viscous damping coefficient as was obtained before by Kimbark's formula.

4.8.1 Machine Windings Representation

The differential equation represent the field winding in this model is the same as Eqn. (4.23) except that E_{FD} is considered as the output of a 1st order excitation system.

$$\Delta \ddot{E}_q' = -\frac{\Delta E_q'}{T_{do}'} - \frac{X_d - X_d'}{T_{do}'} \Delta i_d + \frac{\Delta E_{FD}}{T_{do}'} \tag{4.50}$$

Also the algebraic equations are same as Eqns. (4.24) and (4.25) because the same model is used. So, the phasor diagram describing the algebraic equation is the same as drawn in Fig. 4.8.

4.8.2 Machine Windings Representation Including the Effect of the Infinite Bus and the Transmission Line

It is similarly as did for third order engineering model except that ΔE_{FD} now becomes a state variable. According to this, Eqn. (4.37) is modified to;

$$(X_d' + X_e)\, \Delta \dot{i}_d - V_B \sin \delta_o \, \Delta \dot{\delta} = \frac{X_d + X_e}{T_{do}'} \Delta i_d + \frac{V_B \sin \delta_o}{T_{do}'} \Delta \delta + \frac{\Delta E_{FD}}{T_{do}'} \tag{4.51}$$

4.8.3 Excitation System Representation

A static AVR is derived from the IEEE type 1 [1] excitation system, which is used in the system described in chapter 3. The way to derive this static AVR is based on comparing the different time constants of the individual block of the IEEE type 1 Excitation system. By doing this the effective time constant T_e and the overall gain K_e can be calculated. Another method is to reduce the IEEE type 1 excitation system using the model equivalent approach used in chapter 2 by performing the T-factors for the AVR-excitation system above and then one mode is preserved which is the more controllable and more observable. In our case a fourth order excitation system can be reduced to first order. The fourth order excitation system state variables form is;

$$\dot{X}_e = A_e X_e + B_e u \tag{4.52}$$

where

A_e and B_e are constant system matrices described in Appendix A3-II
$X_e^t = [\Delta E_{FD}\ \Delta V_a\ \Delta V_r\ \Delta V_s]$
$U^t = [\Delta V_t]$

This model can be reduced to first order as

$$\dot{X} = a_{e_1} X_{e_1} + b_{e_1} u \tag{4.53}$$

where

a_{e_1} and b_{e_1} are constants of the reduced system

$X^t_{e_1} = [\Delta E_{FD}]$

a_{e_1} and b_{e_1} are calculated from the model equivalent.

Equation (4.53) can be written as

$$\Delta \dot{E}_{FD} = a_{e_1} \Delta E_{FD} + b_{e_1} \Delta V_t \tag{4.54}$$

Since ΔV_t depends on the terminal generator outputs, it's necessary to express ΔV_t in terms of model variables. Appendix A4-V shows the relation between ΔV_t and the model variables which is,

$$\Delta V_t = k_{12} \Delta i_d + k_{13} \Delta \delta \tag{4.55}$$

where

$$k_{12} = \frac{V_{t_{qo}}}{V_{to}} X_e$$

$$k_{13} = \frac{V_{t_{do}} V_{qo}}{V_{to}} \frac{X_q}{X_q+X_e} - \frac{V_{t_{qo}} V_{do}}{V_{to}}$$

Substituting (4.55) in (4.54)

$$\Delta\dot{E}_{FD} = a_{e_1} \Delta E_{FD} + b_{e_1} k_{12} \Delta i_d + b_{e_1} k_{13} \Delta\delta \qquad (4.56)$$

The overall gain and the time constant of Eqn. (4.56) can be calculated easily from the transfer function of the equivalent system. Since the modal equivalent is 1st order, the reciprocal of the absolute value of the element a_{e_1} is the time constant T_e , and with the known value b_{e_1} the different gains such as k_{12} and k_{13} can be calculated yielding;

$$\Delta\dot{E}_{FD} = \frac{K_{11}}{T_e} \Delta E_{FD} + \frac{K_{12}}{T_e} \Delta i_d + \frac{K_{13}}{T_e} \Delta\delta \qquad (4.57)$$

where

$$K_{11} = a_{e_1} T_e = -1$$

$$K_{12} = b_{e_1} k_{12} T_e$$

$$K_{13} = b_{e_1} k_{13} T_e$$

4.8.4 Shaft System Representation

The shaft is represented by a second order model as did before in section 4.7.3 for a classical third order engineering model. Since there is no change in this model, the differential equation should stay the same as it was for a third order.

$$\Delta\dot{\delta} = \Delta\omega \tag{4.58}$$

$$\Delta\dot{\omega} = -\frac{D}{M}\Delta\omega - \frac{K_{42}}{M}\Delta i_d - \frac{K_{43}}{M}\Delta\delta + \frac{1}{M}\Delta P_L \tag{4.59}$$

and

$$\Delta P_e = K_{42}\Delta i_d + K_{43}\Delta\delta \tag{4.60}$$

where

$$K_{42} = K_{31} = V_{do}$$

$$K_{43} = K_{32} = (V_{qo}\, I_{do} - V_{do}\, I_{qo} + \frac{V_{qo}^2}{X_q + X_e})$$

4.8.5 Complete Generating Unit Model

Equations (4.51), (4.57), (4.58) and (4.59) represent a simplified 4th order model for a generating unit. These equations are summarized in Table 4.6.

Figure 4.13 shows a block diagram of a complete generator unit including the machine winding, the shaft and the excitation system.

TABLE 4.6. Summary of Equations for Describing Classical Fourth Order Engineering Model.

Differential Equations:

$$\Delta \dot{E}_{FD} = \frac{K_{11}}{T_e} \Delta E_{FD} + \frac{K_{12}}{T_e} \Delta i_d + \frac{K_{13}}{T_e} \Delta\delta \quad (4.57)$$

$$(X_d' + X_e)\, \Delta \dot{i}_d - V_B \sin\delta_o \Delta\dot{\delta} = \frac{1}{T_{do}'} \Delta E_{FD} - \frac{X_d + X_e}{T_{do}'} \Delta i_d + \frac{V_B \sin\delta_o}{T_{do}'} \Delta\delta \quad (4.51)$$

$$\Delta\dot{\delta} = \Delta\omega \quad (4.58)$$

$$\Delta\dot{\omega} = -\frac{K_{42}}{M} \Delta i_d - \frac{K_{43}}{M} \Delta\delta - \frac{D}{M} \Delta\omega + \frac{1}{M} \Delta P_L \quad (4.59)$$

Algebraic Equations

$$\Delta P_e = K_{42} \Delta i_d + K_{43} \Delta\delta \quad (4.60)$$

where

$$K_{42} = V_{do}$$

$$K_{43} = (V_{qo} I_{do} - V_{do} I_{qo} + \frac{V_{qo}^2}{X_q + X_e})$$

Units: t in seconds, ω in rad/sec M in p.u. power sec^2, D in p.u. power sec. currents and power in per unit.

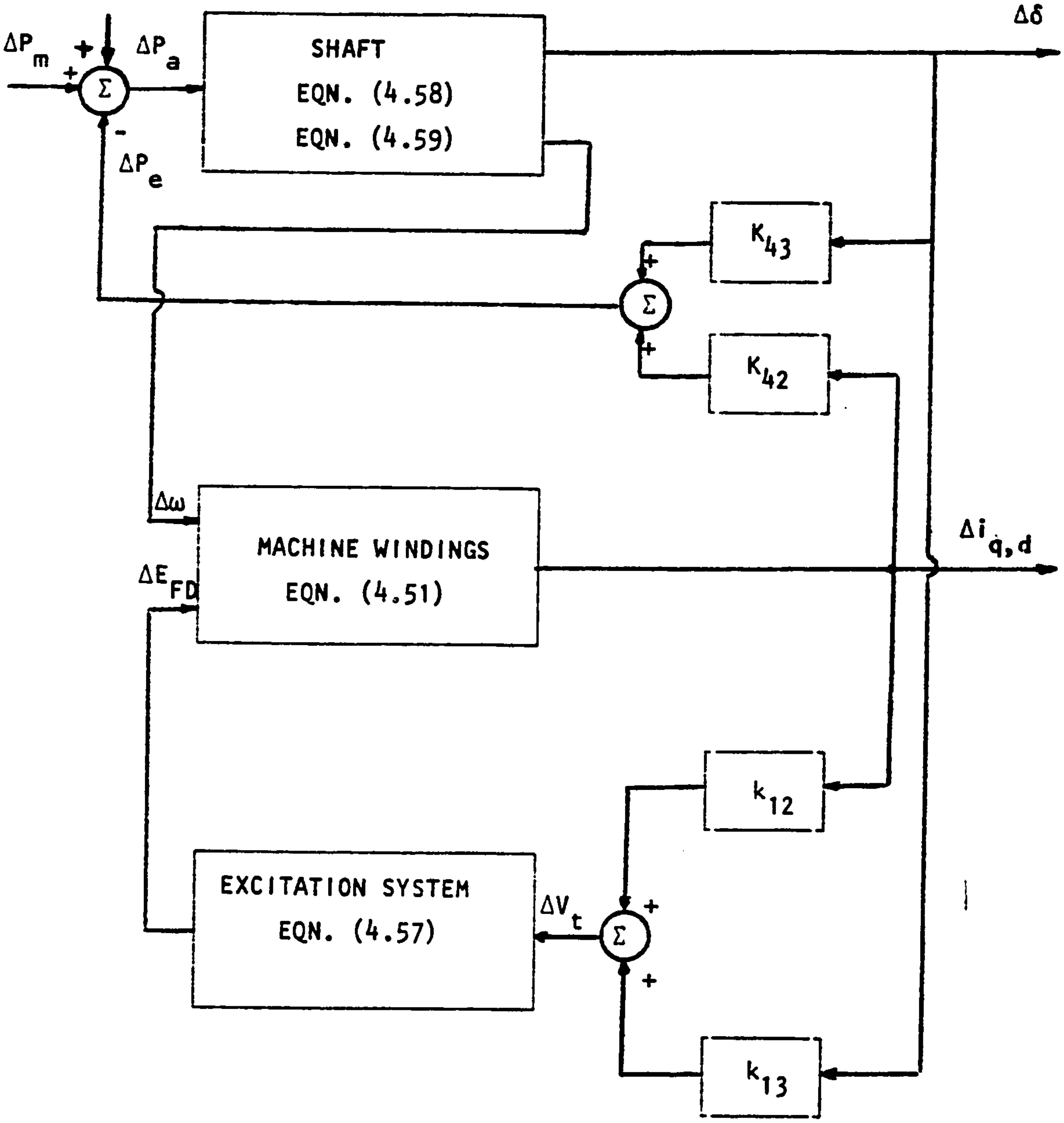


Figure 4.13. Block diagram representation of the classical fourth order Engineering model.

The state variable form can be written as

$$L_4 \dot{X}_4 = Z_4 X_4 + B_4 u \qquad (4.61)$$

where

$$X_4^t = [\ \Delta E_{FD} \quad \Delta i_d \quad \Delta\delta \quad \Delta\omega\ ]$$

$$L_4 = \begin{array}{c} \begin{array}{cccc} \Delta E_{FD} & \Delta i_d & \Delta\delta & \Delta\omega \end{array} \\ \begin{bmatrix} 1 & & & \\ & X_d'+X_e & -V_B \sin\delta_o & \\ & & 1 & \\ & & & 1 \end{bmatrix} \end{array}$$

$$B_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M} \end{bmatrix}$$

$$
Z_4 = \begin{array}{cccc}
\Delta E_{FD} & \Delta i_d & \Delta\delta & \Delta\omega \\
\end{array}
$$

$$
Z_4 = \begin{bmatrix}
\frac{K_{11}}{T_e} & \frac{K_{12}}{T_e} & \frac{K_{13}}{T_e} & 0 \\
\frac{1}{T'_{do}} & \frac{X_d + X_e}{T'_{do}} & \frac{V_B \sin\delta_o}{T'_{do}} & 0 \\
0 & 0 & 0 & 1 \\
0 & -\frac{K_{42}}{M} & -\frac{K_{43}}{M} & -\frac{D}{M}
\end{bmatrix}
$$

Equation (4.61) can be written in the form

$$
\dot{X}_4 = A_4 X_4 + B_4 u \tag{4.62}
$$

where

$$
A_4 = L_4^{-1} Z_4
$$

$$
B_4 = L_4^{-1} B_4 = B_4
$$

where

$$
A_4 = \begin{array}{cccc}
\Delta E_{FD} & \Delta i_d & \Delta\delta & \Delta\omega \\
\end{array}
$$

$$
A_4 = \begin{bmatrix}
\frac{K_{11}}{T_e} & \frac{K_{12}}{T_e} & \frac{K_{13}}{T_e} & 0 \\
\frac{K_{21}}{T'_{do}} & -\frac{K_{22}}{T'_{do}} & \frac{K_{23}}{T'_{do}} & \frac{V_B \operatorname{Sin} \delta_o}{X'_d + X_e} \\
0 & 0 & 0 & 1 \\
0 & -\frac{K_{42}}{M} & -\frac{K_{43}}{M} & -\frac{D}{M}
\end{bmatrix}
$$

All the K's are listed below:

$$K_{11} = a_{e_1} T_e$$

$$K_{12} = b_{e_1} k_{12} T_e$$

$$K_{13} = b_{e_1} k_{13} T_e$$

$$K_{21} = \frac{1}{X'_d + X_e}$$

$$K_{22} = \frac{X_d + X_e}{X'_d + X_e}$$

$$K_{23} = \frac{V_B \sin \delta_o}{X'_d + X_e}$$

$$K_{42} = V_{do}$$

$$K_{43} = (V_{qo} I_{do} - V_{do} I_{qo} + \frac{V^2_{qo}}{X_q + X_e})$$

4.8.6 Results for Reduced Excitation System

The numerical value for the constant matrices of Eqn. (4.52) are

$$A_e = \begin{array}{c} \begin{array}{cccc} \Delta E_{FD} & \Delta V_a & \Delta V_r & \Delta V_s \end{array} \\ \begin{bmatrix} -.4931 & 6.8491 & 0.0 & 0.0 \\ 0.0 & -50 & -2500 & -2500 \\ 0.0 & 0.0 & -1000 & 0.0 \\ -.0625 & .8676 & 0.0 & -2.222 \end{bmatrix} \end{array}$$

$$B_e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1000 \end{bmatrix}$$

The eigenvalues of the matrix A_e are

$$\lambda_1 = -1000$$

$$\lambda_{2,3} = -26.3457 \pm j40.1297$$

$$\lambda_4 = -.0238$$

applying the approach discussed in Chapter 2 for the system described by Eqn. (4.52) for a step change in V_t, the input-output performance indices are as shown in Table 4.7.

According to the input-output performance indices the lowest order for the output ΔE_{FD} is a first order from mode 4 which is the highest T-factor compared to the other modes.

The matrix a_{e_1} and the vector b_{e_1} are single element in this case as

$$a_{e_1} = -.0238$$

$$b_{e_1} = -16.3458$$

If more accuracy is needed a third order from modes 4 and the complex conjugate pair 2, 3 can be done.

TABLE 4.7. The Input-Output Performance Indices to a Step Change in ΔV_t Input and ΔE_{FD} Output.

Mode	Eigenvalue	T
1	-1000	-.02
2	-26.35 - j40.1297	-3.51 - j2.73
3	-26.35 + j40.1297	-3.51 + j2.73
4	-.0238	-685.81

Table 4.8 shows the p.u. R.M.S. error of the exciter reduced model which is calculated over a 100 second period using the formula derived in Chapter 2.

The output of the excitation system (b) reduced order model is calculated and plotted versus the complete response (a) of 4th order model to a 1.0% step change in the input voltage V_t, also the output error of the reduced model is plotted below the output over the same studied period in Figs. 4.14 and 4.15 respectively.

4.8.7 Result of Classical Fourth Order Model

Using the data and the operating point calculated in section 4.3, the numerical values for the constants K's are be calculated as follows.

$k_{12} = .1821$ p.u.

$k_{13} = -.18295$ p.u.

$K_{21} = 2.673968$ p.u.

$K_{22} = 4.95455$ p.u.

$K_{23} = 1.506754$ p.u.

$K_{42} = .564$ p.u.

$K_{43} = .9849$ p.u.

$D = .092701$ p.u.

$a_{e_1} = -.02383$ p.u.

$b_{e_1} = -16.3458$ p.u.

TABLE 4.8. Per Unit R.M.S. Error of the Reduced Excitation System.

Order of model	E_{FD}	Preserved Eigenvalue
1	.015151	4

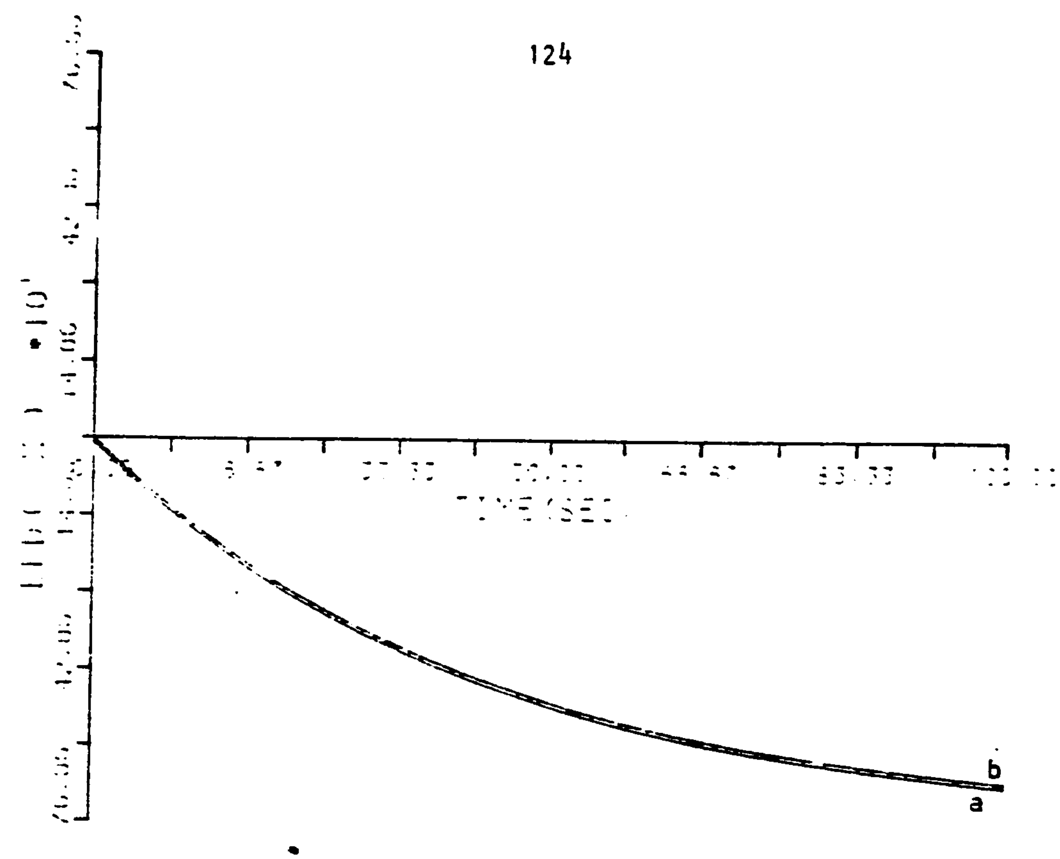


Figure 4.14. Exciter output voltage response to a 1.0% step charge in V_t.

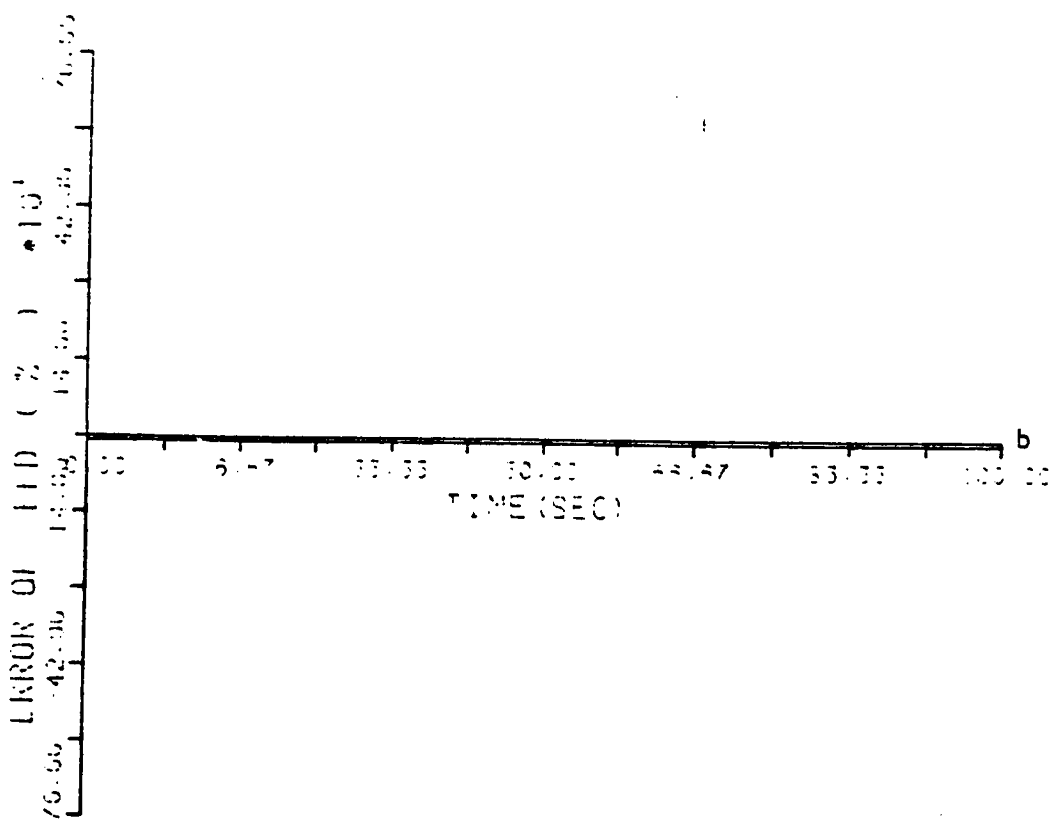


Figure 4.15. Exciter output voltage error to a 1.0% step change in V_t.

$T_e = \frac{1}{|-.02383|} = 41.964$ p.u.

$K_{11} = -1$

$K_{12} = b_{e_1} k_{12} T_e = -124.9$

$K_{13} = b_{e_1} k_{13} T_e = 125.5$

Using the values of the K's calculated above and using the typical parameters of the generating unit the matrix A_4 can be calculated as follows

$$A_4 = \begin{array}{c} \begin{array}{cccc} \Delta E_{FD} & \Delta i_d & \Delta\delta & \Delta\omega \end{array} \\ \begin{pmatrix} -.02383 & -2.976 & 2.991 & 0 \\ .71190 & -1.13364 & .34479 & 1.50675 \\ & & & 1 \\ 0 & -40.28 & -70.3502 & -6.6215 \end{pmatrix} \end{array}$$

The eigenvalues of the matrix A_4 are

$\lambda_{1,2} - .367 \pm j1.179$

$\lambda_{3,4} -3.518 \pm j11.022$

However, the eigenvalues of the reduced fourth order model are

$\lambda^*_{1,2} = -.661 \pm j1.018$

$\lambda^*_{3,4} = -2.574 \pm j12.854$

Table 4.9 summarizes the results of the reduced fourth order model and the classical fourth order Engineering model showing the eigenvalues and the p.u. R.M.S. error.

As a conclusion, the results obtained show that the accuracy for the classical fourth order Engineering model is more accurate than the reduced fourth order and this indicated that there is no need to compensate the parameters of the synchronous machine.

The output of (a) the reduced fourth order model, and (b) the classical fourth order model are calculated and plotted versus the complete response of the 13th order model to a 1% step change in the load power. Also the error of these models are calculated w.r.t. 13th order model and plotted below each output over the same studied period in Figs. 4.16 through 4.23.

4.9 CLASSICAL FIFTH ORDER ENGINEERING MODEL

Our objective in this model is to investigate the effect of the damper winding in the performance of the generating unit. Theoretically the damper winding plays an important part in the damping of the synchronous machine. In this model we add a new state variable to the model described in section 4.7. This variable is the voltage E_d' which is a function of the q-axis damper circuit current. This voltage caused by the flux linkage induced by the damper winding circuit in the quadrature axis. Generally, the transient effects are dominated by two rotor circuits, which are the field circuit in the

TABLE 4.9. Summary of the Results Obtained for Fourth Order Model.

	Model	Reduced(1)	Classical
Eigenvalues	$\lambda_{1,2}$	$-2.574 \pm j12.854$	$-3.518 \pm j11.022$
	$\lambda_{3,4}$	$-.661 \pm j1.018$	$-.367 \pm j1.179$
p.u.(2) R.M.S. ERROR	δ	.0737	.0625
	ω	.7712	.361
	I_d	.198	.1
	E_{fd}	.545	.2794
	I_q	----	----

(1) Reduced from 13th order model

(2) p.u. R.M.S. Error $= [\sum_{t_1}^{t_2} (x - x^*)^2/x^2]^{1/2}$

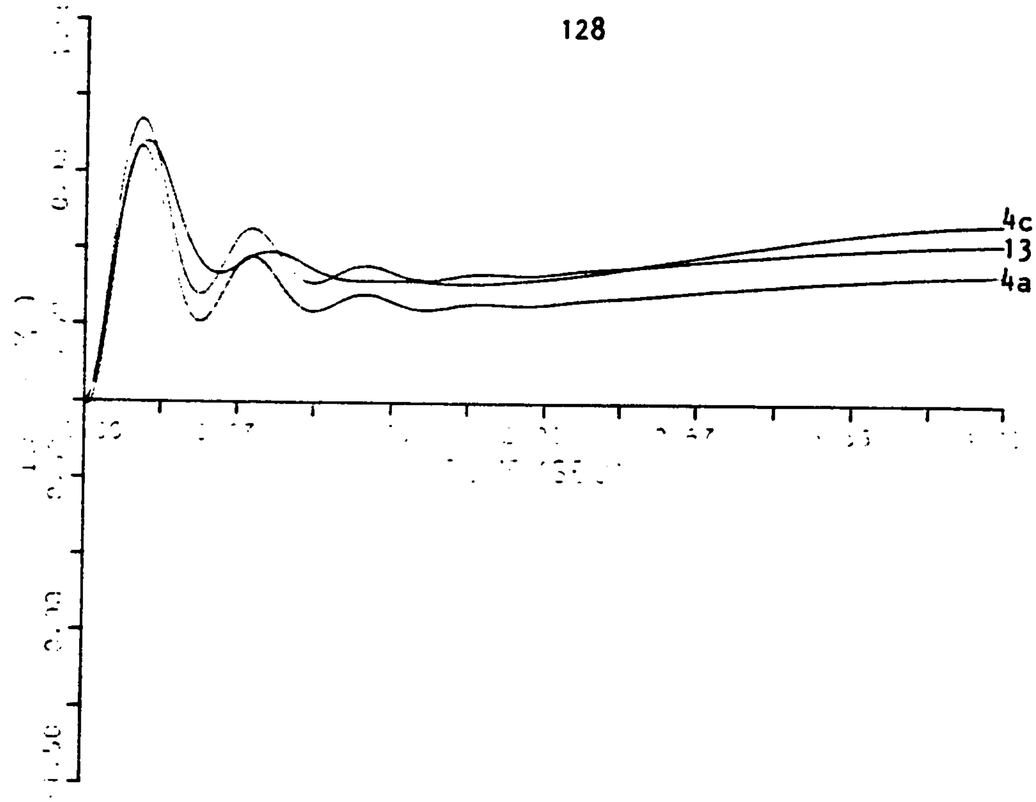


Figure 4.16. I_d response to a 1.0% step change in P_L.

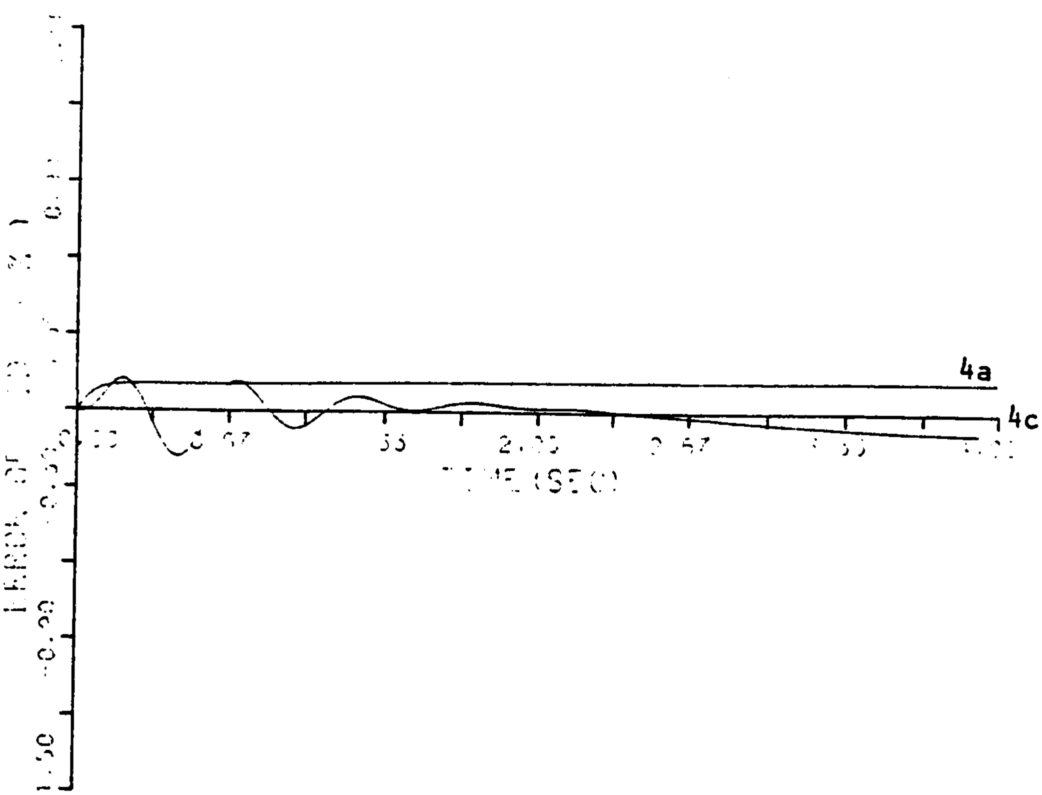


Figure 4.17. I_d error to a 1.0% step change in P_L.

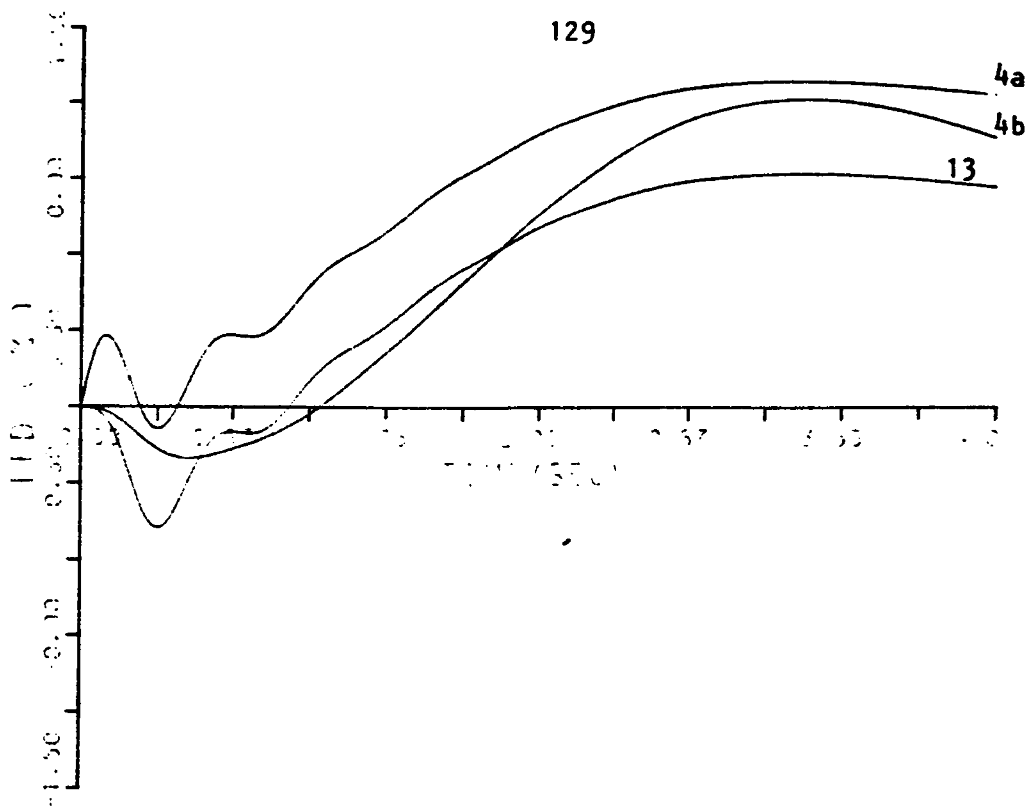


Figure 4.18. Exciter output voltage response to a 1.0% step change in P_L.

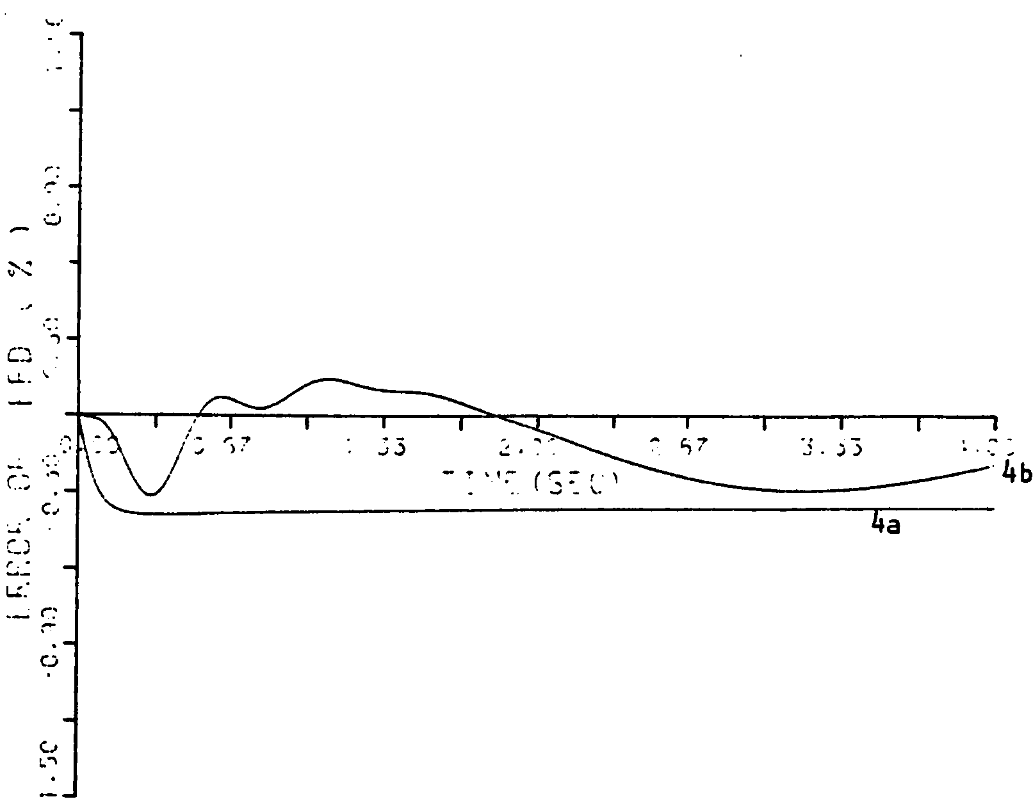


Figure 4.19. Exciter output voltage error to a 1.0% step change in P_L.

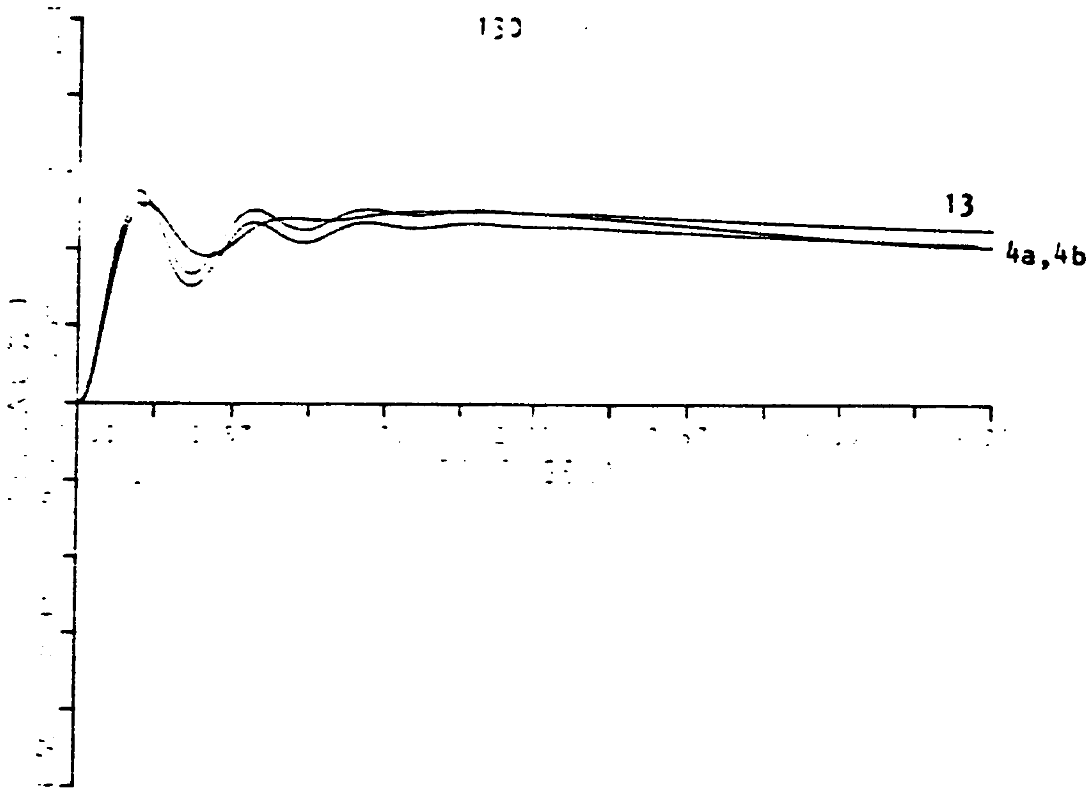


Figure 4.20. Rotor angle response to a 1.0% step change in P_L.

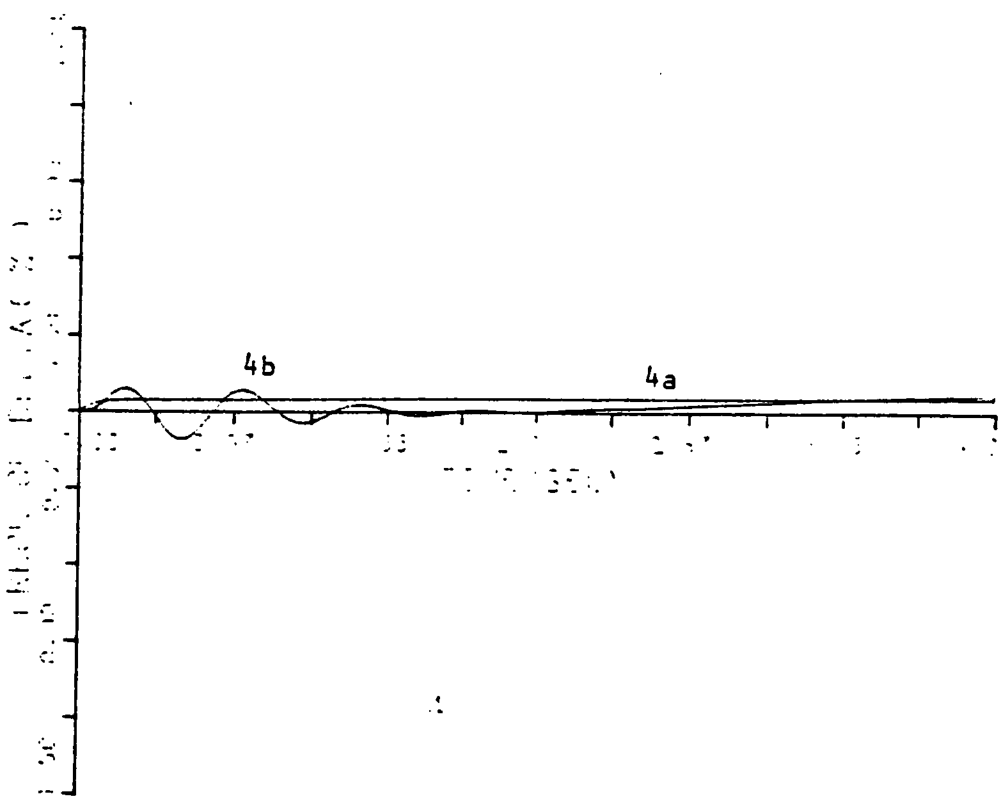


Figure 4.21. Rotor angle error to a 1.0% step change in P_L.

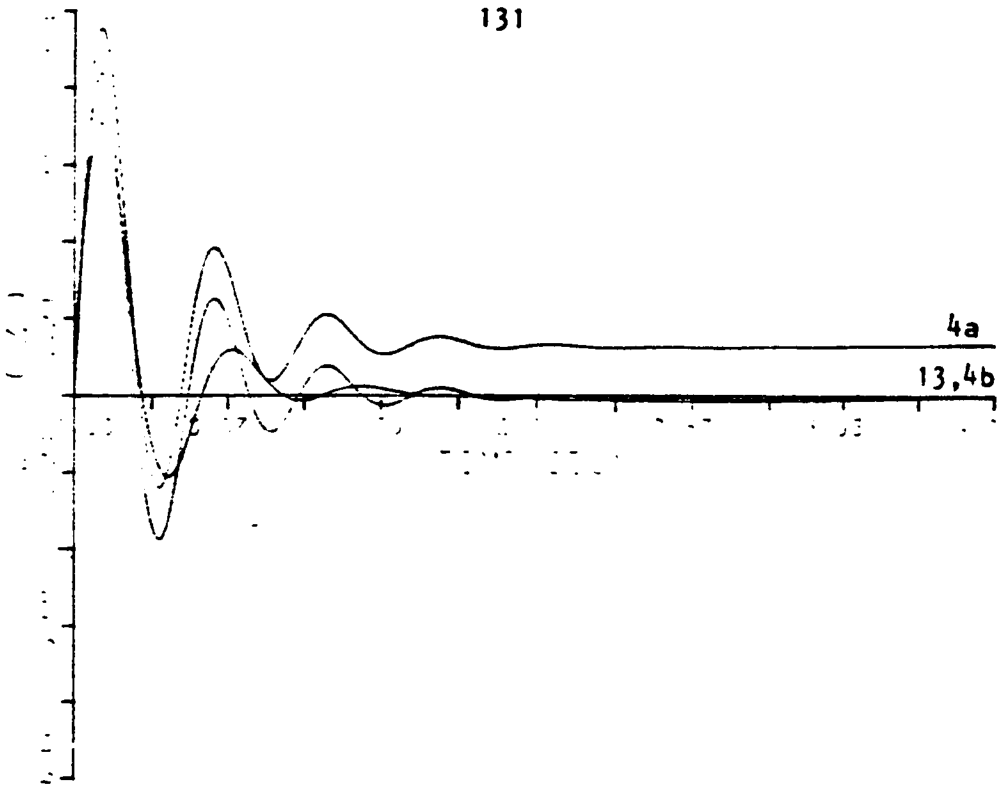


Figure 4.22. Rotor frequency response to a 1.0% step change in P_L.

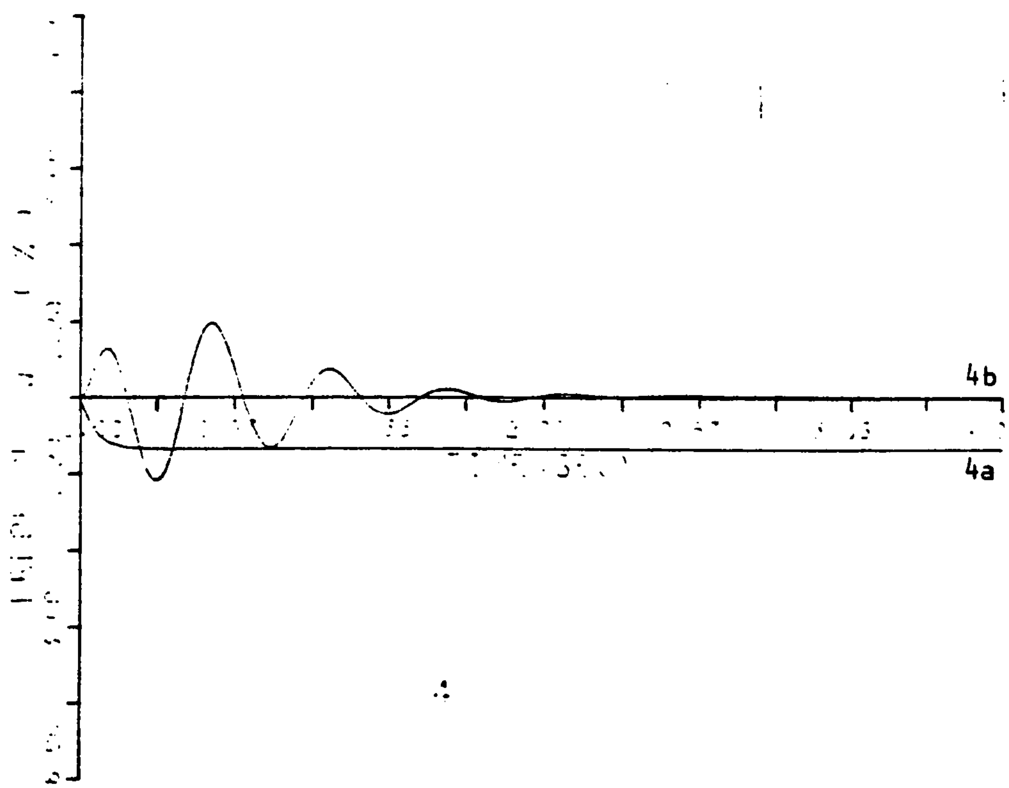


Figure 4.23. Rotor frequency error to a 1.0% step change in P_L.

d axis, and a single damper winding circuit in the q axis or an equivalent circuit in the q axis formed by the solid rotor if non-salient pole synchronous machine.

Now, the machine windings are represented by a second order, the exciter is represented by a first order and the shaft is represented by a second order. Still in this model the governor/turbine system is neglected.

4.9.1 Machine Windings Representation

Appendix A4-I shows a detailed derivation for the machine windings including field circuit and single damper winding circuit in the quadrature axis. The differential equations are

$$\Delta \dot{E}_q' = -\frac{\Delta E_q'}{T_{do}'} - \frac{X_d - X_d'}{T_{do}'} \Delta i_d + \frac{\Delta E_{FD}}{T_{do}'} \qquad (4.63)$$

$$\Delta \dot{E}_d' = -\frac{\Delta E_d'}{T_{qo}'} + \frac{X_q - X_q'}{T_{qo}'} \Delta i_q \qquad (4.64)$$

and the algebraic equations are the same as Eqns. (4.24) and (4.25) except that Eqn. (4.25) is modified to:

$$\Delta V_d = \Delta E_d' + X_q' \Delta i_q \qquad (4.65)$$

ΔV_q and ΔV_d can be expressed in terms of the rotor angle. The algebraic equations can be written in the form

$$\Delta \dot{E}'_q = -V_B \sin \delta_o \, \Delta\delta + \dot{X}'_d \, \Delta i_d \quad (4.66)$$

$$\Delta \dot{E}'_d = V_B \cos \delta_o \, \Delta\delta - \dot{X}'_q \, \Delta i_q \quad (4.67)$$

4.9.2 Machine Windings Representation Including the Infinite Bus and Transmission Line

As was did previously the reactances of the direct and quadrature axis are modified such as

$$\hat{X}_d = X_d + X_e \quad (4.68)$$

$$\hat{X}_q = X_q + X_e \quad (4.69)$$

and by the same way the transient and subtransient reactances can be modified. Also the algebraic equations are modified to,

$$\Delta \dot{E}'_q = -V_B \sin \delta_o \, \Delta\delta + (\dot{X}'_d + X_e) \, \Delta i_d \quad (4.70)$$

$$\Delta \dot{E}'_d = -V_B \sin \delta_o \, \Delta\delta + (\dot{X}'_q + X_e) \, \Delta i_q \quad (4.71)$$

Equations (4.63) and (4.64) can be written in terms of Δi_q and Δi_d as state variables instead of $\Delta E_q'$ and $\Delta E_d'$.

$$-(X_q'+X_e)\,\Delta\dot{i}_q+V_B\cos\delta_o\Delta\dot{\delta} = \frac{(X_q'+X_e)}{T_{qo}'}\Delta i_q - \frac{V_B\cos\delta_o}{T_{qo}'}\Delta\delta + \frac{X_q-X_q'}{T_{qo}'}\Delta i_q \tag{4.72}$$

rearranging (4.72)

$$-(X_q'+X_e)\,\Delta\dot{i}_q+V_B\cos\delta_o\Delta\dot{\delta} = \frac{(X_q+X_e)}{T_{qo}'}\Delta i_q - \frac{V_B\cos\delta_o}{T_{qo}'}\Delta\delta \tag{4.73}$$

and,

$$(X_d'+X_e)\,\Delta\dot{i}_d-V_B\sin\delta_o\Delta\dot{\delta} = \frac{\Delta E_{FD}}{T_{do}'} - \frac{X_d'+X_e}{T_{do}'}\Delta i_d + \frac{V_B\sin\delta_o}{T_{do}'}\Delta\delta + \frac{X_d'-X_d}{T_{do}'}\Delta i_d \tag{4.74}$$

rearranging (4.74)

$$(X_d'+X_e)\,\Delta\dot{i}_d-V_B\sin\delta_o\Delta\dot{\delta} = \frac{\Delta E_{FD}}{T_{do}'} - \frac{X_d+X_e}{T_{do}'}\Delta i_d + \frac{V_B\sin\delta_o}{T_{do}'}\Delta\delta \tag{4.75}$$

4.9.3 Excitation System Representation

It is similar to the type discussed in section 4.8.3 except that ΔV_t in this model is a function of the direct and quadrature axis currents and the rotor angle. Appendix A4-V derive the relation between ΔV_t and the model variables.

$$\Delta V_t = k_{12}\Delta i_q + k_{13}\Delta i_d + k_{14}\,\Delta\delta \tag{4.76}$$

where

$$k_{12} = -\frac{V_{t_{do}}}{V_{to}} X_e$$

$$k_{13} = \frac{V_{t_{qo}}}{V_{to}} X_e$$

$$k_{14} = \frac{V_{t_{do}}}{V_{to}} V_B \cos\delta_o - \frac{V_{t_{qo}}}{V_{to}} V_B \sin\delta_o$$

Following the same procedure as was done in section 4.8.3 we can end up with:

$$\Delta\dot{E}_{FD} = \frac{K_{11}}{T_e}\,\Delta E_{FD} + \frac{K_{12}}{T_e}\,\Delta i_q + \frac{K_{13}}{T_e}\,\Delta i_d + \frac{K_{14}}{T_e}\,\Delta\delta \tag{4.77}$$

where

$$K_{11} = a_{e_1} T_e$$

$$K_{12} = b_{e_1} k_{12} T_e$$

$$K_{13} = b_{e_1} k_{13} T_e$$

$$K_{14} = b_{e_1} k_{14} T_e$$

4.9.4 Shaft System Representation

The electrical power is

$$P_e = V_q i_q + V_d i_d \tag{4.78}$$

Substituting for V_q and V_d

$$P_e = V_B \cos\delta\, i_q + V_B \sin\delta\, i_d \tag{4.79}$$

Differentiating Eqn. (4.79)

$$\Delta P_e = V_B \cos\delta_o \Delta i_q - V_B I_{qo} \sin\delta_o \Delta\delta + V_B \sin\delta_o \Delta i_d + V_B I_{do} \cos\delta_o \Delta\delta \tag{4.80}$$

rearranging Eqn. (4.80)

$$\Delta P_e = K_{52}\Delta i_q + K_{53}\Delta i_d + K_{54}\,\Delta\delta \tag{4.81}$$

where

$$K_{52} = V_B \cos\delta_o = V_{qo}$$

$$K_{53} = V_B \sin\delta_o = V_{do}$$

$$\begin{aligned} K_{54} &= V_B I_{do} \cos\delta_o - V_B I_{qo} \sin\delta_o \\ &= V_{qo} I_{do} - V_{do} I_{qo} \end{aligned}$$

The differential equations describing the shaft model are

$$\Delta\dot{\delta} = \Delta\omega \tag{4.82}$$

$$\Delta\dot{\omega} = -\frac{D}{M}\,\Delta\omega - \frac{K_{52}}{M}\,\Delta i_q - \frac{K_{53}}{M}\,\Delta i_d - \frac{K_{54}}{M}\,\Delta\delta + \frac{1}{M}\,\Delta P_L \tag{4.83}$$

4.9.5 Complete Generating Unit Model

Equations (4.77), (4.73), (4.75), (4.82) and (4.83) represent a simplified fifth order model for a generating unit. These equations are summarized in Table 4.10.

TABLE 4.10. Summary of Equations for Describing a Classical Fifth Order Engineering Model.

Differential Equations

$$\Delta\dot{E}_{FD} = \frac{K_{11}}{T_e}\Delta E_{FD} + \frac{K_{12}}{T_e}\Delta i_q + \frac{K_{13}}{T_e}\Delta i_d + \frac{K_{14}}{T_e}\Delta\delta \qquad (4.77)$$

$$-(X_q' + X_e)\,\Delta\dot{i}_q + V_B\cos\delta_o\Delta\dot{\delta} = \frac{X_q + X_e}{T_{qo}'}\Delta i_q - \frac{V_B\cos\delta_o}{T_{qo}'}\Delta\delta \qquad (4.73)$$

$$(X_d' + X_e)\,\Delta\dot{i}_d - V_B\sin\delta_o\Delta\dot{\delta} = \frac{\Delta E_{FD}}{T_{do}'} - \frac{X_d + X_e}{T_{do}'}\Delta i_d + \frac{V_B\sin\delta_o}{T_{do}'}\Delta\delta \qquad (4.75)$$

$$\Delta\dot{\delta} = \Delta\omega \qquad (4.82)$$

$$\Delta\dot{\omega} = -\frac{K_{52}}{M}\Delta i_q - \frac{K_{53}}{M}\Delta i_d - \frac{K_{54}}{M}\Delta\delta - \frac{D}{M}\Delta\omega + \frac{1}{M}\Delta P_L \qquad (4.83)$$

Algebraic Equations

$$\Delta P_e = K_{52}\Delta i_q + K_{53}\Delta i_d + K_{54}\Delta\delta \qquad (4.81)$$

$K_{52} = V_{qo}$

$K_{53} = V_{do}$

$K_{54} = V_{qo}I_{do} - V_{do}I_{qo}$

Units

t in second, ω in rad/sec, M in p.u. power sec^2, D in p.u. power sec, currents and power in per unit.

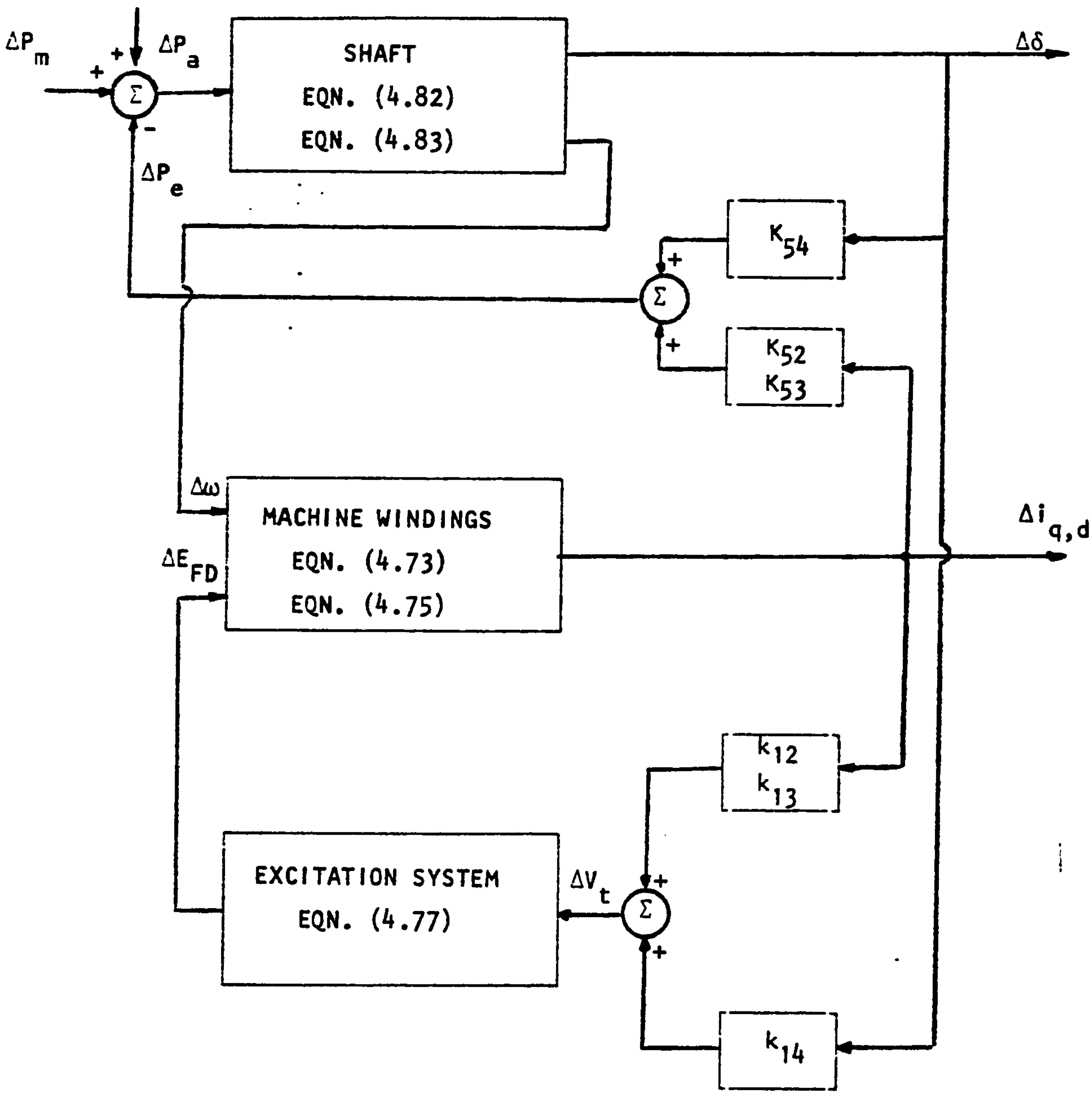


Figure 4.24. Block diagram representation of the classical fifth order Engineering model.

Figure 4.24 shows a block diagram of a complete generating unit including the machine winding, the shaft and the excitation system.

The state variable equation form for this model can be written as

$$L_5 \dot{X}_5 = Z_5 X_5 + B_5 U \tag{4.84}$$

where

$$X_5 = [\, \Delta E_{FD} \quad \Delta i_q \quad \Delta i_d \quad \Delta\delta \quad \Delta\omega \,]$$

$$L_5 = \begin{array}{c} \begin{matrix} \Delta E_{FD} & \Delta i_q & \Delta i_d & \Delta\delta & \Delta\omega \end{matrix} \\ \begin{bmatrix} 1 & & & & \\ & -(X_q'+X_e) & & V_B \operatorname{Cos}\delta_o & \\ & & X_d'+X_e & -V_B \operatorname{Sin}\delta_o & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \end{array}$$

$$Z_5 = \begin{array}{c} \begin{matrix} \quad \Delta E_{FD} & \quad \Delta i_q & \quad \Delta i_d & \quad \Delta\delta & \quad \Delta\omega \end{matrix} \\ \begin{bmatrix} \frac{K_{11}}{T_e} & \frac{K_{12}}{T_e} & \frac{K_{13}}{T_e} & \frac{K_{14}}{T_e} & 0 \\ 0 & \frac{X_q+X_e}{T'_{qo}} & 0 & -\frac{V_B \operatorname{Cos}\delta_o}{T'_{qo}} & 0 \\ \frac{1}{T'_{do}} & 0 & -\frac{X_d+X_e}{T'_{do}} & \frac{V_B \operatorname{Sin}\delta_o}{T'_{do}} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{K_{52}}{M} & -\frac{K_{53}}{M} & -\frac{K_{54}}{M} & -\frac{D}{M} \end{bmatrix} \end{array}$$

$$B_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{M} \end{bmatrix}$$

After inverting L_5, Eqn. (4.84) becomes

$$\dot{X}_5 = A_5 X_5 + B_5 U \qquad (4.85)$$

where

$A_5 = L_5^{-1} Z_5$

B_5 still the same because all elements of column 5 in L_5^{-1} are zero except the element 5x5 which is unity.

and

$$A_5 = \begin{pmatrix} \frac{K_{11}}{T_e} & \frac{K_{12}}{T_e} & \frac{K_{13}}{T_e} & \frac{K_{14}}{T_e} & 0 \\ 0 & -\frac{K_{22}}{T'_{qo}} & 0 & \frac{K_{24}}{T'_{qo}} & \frac{V_B \cos \delta_o}{X'_q + X_e} \\ \frac{K_{31}}{T'_{do}} & 0 & -\frac{K_{33}}{T'_{do}} & \frac{K_{34}}{T'_{do}} & \frac{V_B \sin \delta_o}{X'_d + X_e} \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{K_{52}}{M} & -\frac{K_{53}}{M} & -\frac{K_{54}}{M} & -\frac{D}{M} \end{pmatrix}$$

where

$$K_{11} = a_{e_1} T_e$$

$$K_{12} = b_{e_1} k_{12} T_e$$

$$K_{13} = b_{e_1} k_{13} T_e$$

$$K_{14} = b_{e_1} k_{14} T_e$$

$$K_{22} = \frac{X_q + X_e}{X_q' + X_e}$$

$$K_{24} = \frac{V_B \cos \delta_o}{X_q' + X_e}$$

$$K_{31} = \frac{1}{X_d' + X_e}$$

$$K_{33} = \frac{X_d + X_e}{X_d' + X_e}$$

$$K_{34} = \frac{V_B \sin \delta_o}{X_d' + X_e}$$

$$K_{52} = V_{qo}$$

$$K_{53} = V_{do}$$

$$K_{54} = V_{qo} I_{do} - V_{do} I_{qo}$$

4.9.6 Results of Reduced Excitation System

Same result as section 4.8.6 (i.e. same a_{e_1} and b_{e_1}) except that the terminal voltage now is a function of quadrature axis current, the direct axis current and the rotor angle.

4.9.7 Results of Classical Fifth Order Model

Using the parameters of the generating unit and the operating point calculated in section 4.3. The numerical value for the K's are calculated as follows:

$k_{12} = -.09$ p.u.

$k_{13} = .18214$ p.u.

$k_{14} = -.1407$ p.u.

$K_{11} = -1$ p.u.

$K_{12} = 61.73$ p.u.

$K_{13} = -124.94$ p.u.

$K_{14} = 96.168$ p.u.

$K_{22} = 5.4414$ p.u.

$K_{24} = 2.54968$ p.u.

$K_{31} = 2.6738$ p.u.

$K_{33} = 4.9545$ p.u.

$K_{34} = 1.50675$ p.u.

$K_{52} = 0.82609$ p.u.

$K_{53} = 0.5635$ p.u.

$K_{54} = 0.6$ p.u.

Using the values of K's calculated above and using the typical parameters of the generating unit the matrix A_5 is calculated as follows:

$$A_5 = \begin{array}{c} \begin{array}{ccccc} \Delta E_{FD} & \Delta i_q & \Delta i_d & \Delta\delta & \Delta\omega \end{array} \\ \left[\begin{array}{ccccc} -.02383 & 1.4711 & -2.977 & 2.292 & 0 \\ 0 & -19.093 & 0 & 8.946 & 2.54968 \\ .61185 & 0 & -1.13375 & .34479 & 1.50675 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -59.006 & -40.25 & -42.857 & 0 \end{array} \right] \end{array}$$

The eigenvalues of this model are

$\lambda_{1,2} = -.384 \pm j1.175$

$\lambda_{3,4} = -2.802 \pm j13.257$

$\lambda_5 = -13.877$

4.10 COMPENSATED CLASSICAL FIFTH ORDER MODEL

The parameters of this model are the inertia M, Damping D, the d-axis open circuit transient time constant T'_{do}, the q-axis open circuit transient time constant T'_{qo} and the main exciter time constant T_e. Applying the same procedure as was done for the previous model.

$$T'^{\,new}_{qo} = T'_{qo} + \Delta T'_{qo} \tag{4.86}$$

$$T'^{\,new}_{do} = T'_{do} + \Delta T'_{do} \tag{4.87}$$

$$M^{new} = M + \Delta M \tag{4.88}$$

$$D^{new} = D + \Delta D \tag{4.89}$$

Substituting (4.86) - (4.89) into the elements of the matrix A_5 on page 80 and equating each element with the corresponding one of the matrix A_5^* obtained in Chapter 3 using the mathematical reduction method.

Then

$$\frac{K_{22}}{T'^{\,new}_{qo}} = A^*_{5_{22}} \tag{4.90}$$

$$\frac{K_{24}}{T'^{\,new}_{qo}} = A^*_{5_{24}} \tag{4.91}$$

$$\frac{K_{31}}{T_{do}^{'\,new}} = A^{*}_{5_{31}} \qquad (4.92)$$

$$-\frac{K_{33}}{T_{do}^{'\,new}} = A^{*}_{5_{32}} \qquad (4.93)$$

$$\frac{K_{34}}{T_{do}^{'\,new}} = A^{*}_{5_{34}} \qquad (4.94)$$

$$-\frac{K_{52}}{M^{new}} = A^{*}_{5_{52}} \qquad (4.95)$$

$$-\frac{K_{53}}{M^{new}} = A^{*}_{5_{53}} \qquad (4.96)$$

$$-\frac{K_{54}}{M^{new}} = A^{*}_{5_{54}} \qquad (4.97)$$

and

$$-\frac{D^{new}}{M^{new}} = A^{*}_{5_{55}} \qquad (4.98)$$

The numerical values of A^{*}_{5} are

$$
\begin{array}{cccccc}
 & \Delta E_{FD} & \Delta i_q & \Delta i_d & \Delta\delta & \Delta\omega \\
A_5^* = & \multicolumn{5}{c}{\left[\begin{array}{ccccc}
-.8328 & -15.7788 & -.5558 & 11.5646 & 1.1804 \\
.04714 & -19.525 & -.18939 & 8.5645 & 2.54968 \\
.59051 & 1.02285 & -1.16681 & -.3269 & 1.50675 \\
0 & 0 & 0 & 0 & 1 \\
.4714 & -49.8514 & -40.561 & -44.375 & -.08558
\end{array}\right]}
\end{array}
$$

The eigenvalues of this model are

$\lambda_1 = -15.139$

$\lambda_{2,3} = -.661 \pm j1.018$

$\lambda_{4,5} = -2.574 \pm j12.854$

Applying the best-fit method explained in Appendix A4-III, the new value for the parameters can be calculated as follows

$$
M^{new} = \frac{K_{52}\, A^*_{5_{52}} + K_{53}\, A^*_{5_{52}} + K_{54}\, A^*_{5_{54}}}{A^{*\,2}_{5_{52}} + A^{*\,2}_{5_{53}} + A^{*\,2}_{5_{54}}} \qquad (4.99)
$$

$$D^{new} = -A^*_{5_{55}} M^{new} \tag{4.100}$$

$$T'^{\,new}_{do} = \frac{K_{31}A^*_{5_{31}} + K_{31}A^*_{5_{33}} + K_{34}A^*_{5_{34}}}{A^{*2}_{5_{31}} + A^{*2}_{5_{33}} + A^{*2}_{5_{34}}} \tag{4.101}$$

$$T'^{\,new}_{qo} = \frac{K_{22}A^*_{5_{22}} + K_{24}A^*_{5_{24}}}{A^{*2}_{5_{22}} + A^*_{5_{24}}} \tag{4.102}$$

Substituting these known values considering the sign of K's, the new parameters are calculated as follows:

$$M^{new} = .01486$$

$$D^{new} = .0012717$$

$$T'^{\,new}_{do} = 3.77945$$

$$T'^{\,new}_{qo} = .281$$

and

$$\Delta M = .0008639$$

$$\Delta D = .0012717$$

$$\Delta T'_{do} = -.59054$$

$$\Delta T'_{qo} = -.0032429$$

Substituting these new parameters in the A matrix of the model given we get

$$A_5^{new} = \begin{array}{ccccc} \Delta E_{FD} & \Delta i_q & \Delta i_d & \Delta\delta & \Delta\omega \end{array}$$

$$A_5^{new} = \begin{bmatrix} -.0238 & 1.4711 & -2.977 & 2.292 & 0 \\ 0 & -19.3644 & 0 & 9.0735 & 2.54968 \\ .7075 & 0 & -1.31091 & .398669 & 1.50675 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -55.592 & -37.92 & -40.377 & -.08558 \end{bmatrix}$$

The eigenvalues of this model are

$\lambda_1 = -14.354$

$\lambda_{2,3} = -.441 \pm j1.253$

$\lambda_{4,5} = -2.775 \pm j12.741$

Tables 4.11 summarizes the results of the reduced fifth, the classical fifth, and the compensated classical fifth order model showing the eigenvalues, the p.u. R.M.S. error, the R.M.S. error and the compensated value for the parameters. As a conclusion, the results of the compensated fifth order model and the classical fifth order model are about the same order which imply that a classical fifth order Engineering model is an acceptable model for studying the behaviour of the hunting frequency.

The output of (a) the reduced fifth order model, (b) the classical fifth order model and (c) the compensated classical fifth order model are calculated and plotted versus the response of the 13th order model for a 1% step change in the load power. Also the errors of these model are calculated w.r.t. 13th order model and plotted below each output over the same studied period in Figs. 4.25 - 4.34.

TABLE 4.11. Summary of the Results Obtained for Fifth Order Model.

Model		Reduced(1)	Classical	Compensated(2) Classical
Eigenvalues	$\lambda_{1,2}$	$-2.574 \pm j12.854$	$-2.802 \pm j13.26$	$-2.775 \pm j12.741$
	$\lambda_{3,4}$	$-.661 \pm j1.018$	$-.384 \pm j1.175$	$-.441 \pm j1.253$
	λ_5	-15.139	-13.877	-14.354
p.u.(3) R.M.S. ERROR	δ	.006	.0455	.042
	ω	.043	.091	.07
	I_d	.033	.0567	.119
	E_{fd}	.0927	.25	.29
	I_q	.0156	.0715	.0511

(1) Reduced from 13th order model

(2) $\Delta M = .0008639$ $\quad \Delta D = .0012717$

$\Delta T'_{d0} = -.590547$ $\quad \Delta T_{q0} = -.0032429$

(3) p.u. R.M.S. Error = $\left[\sum_{t_1}^{t_2} (x - x^*)^2 / x^2\right]^{1/2}$

360

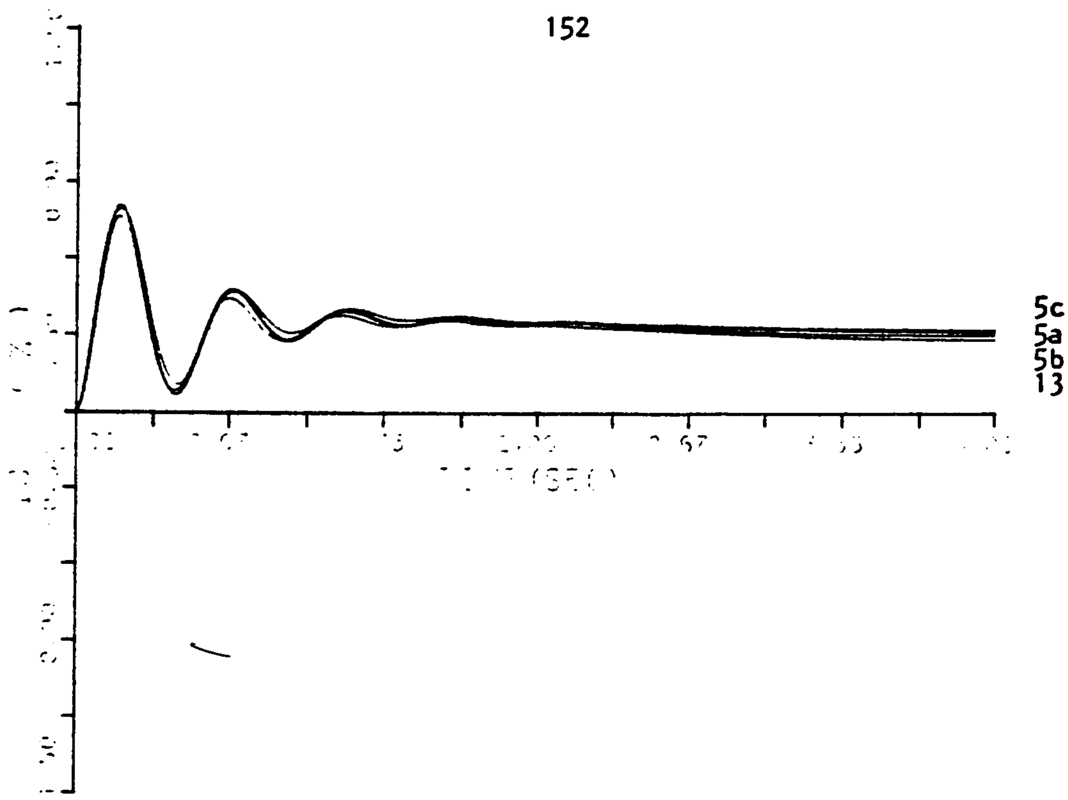


Figure 4.25. I_q response to a 1.0 step change in P_L.

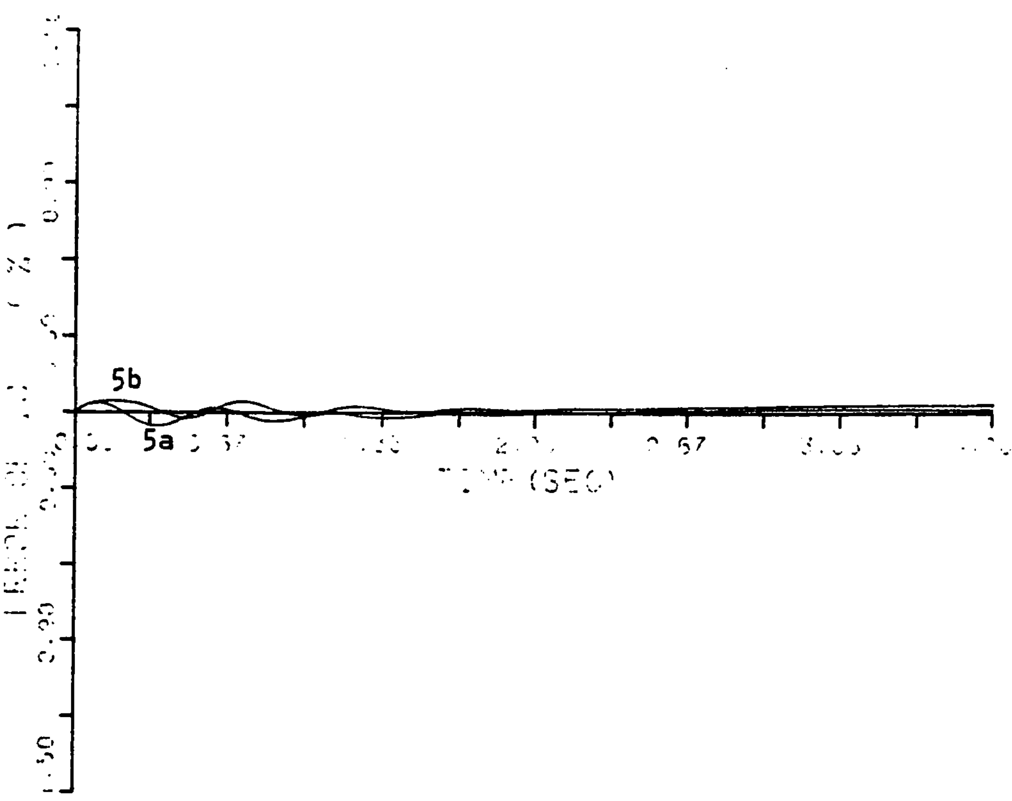


Figure 4.26. I_q error to a 1.0% step change in P_L.

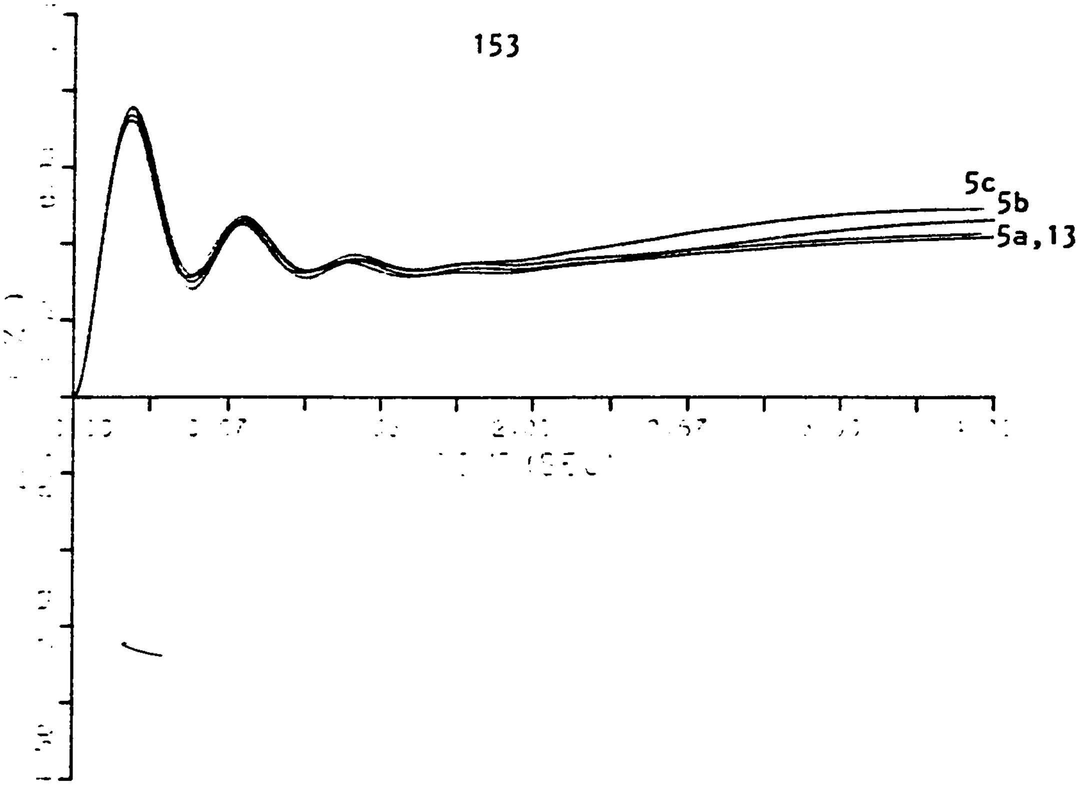


Figure 4.27. I_d response to a 1.0% step change in P_L.

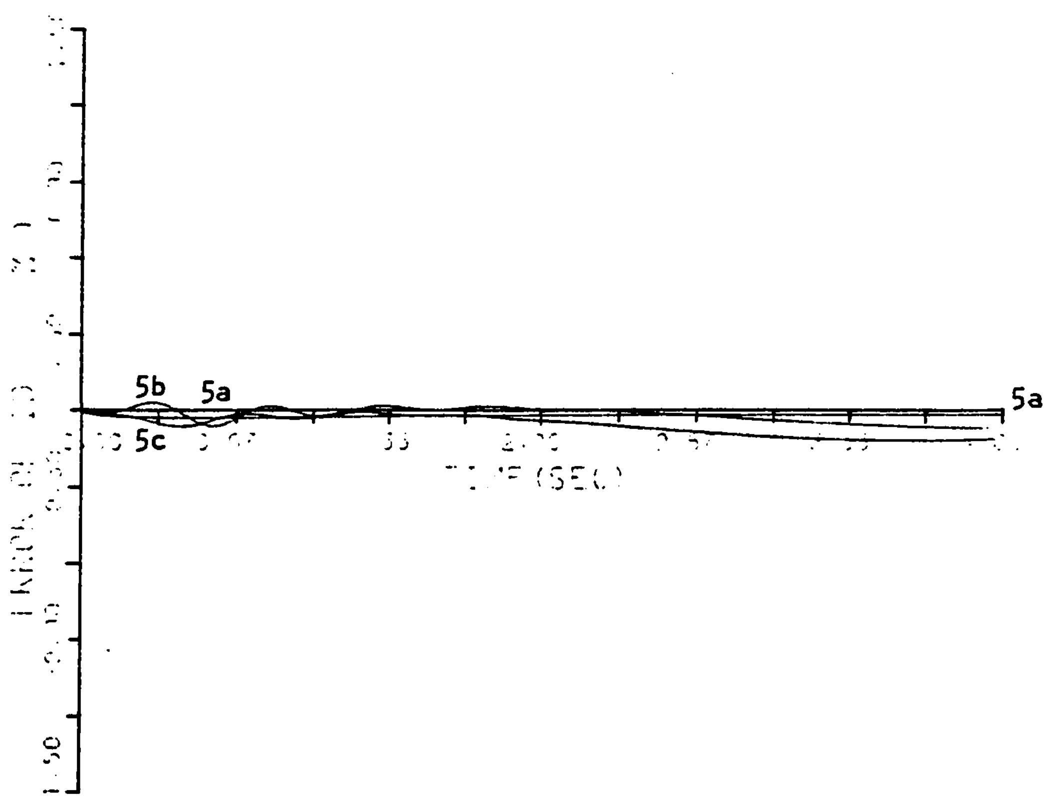


Figure 4.28. I_d error to a 1.0% step change in P_L.

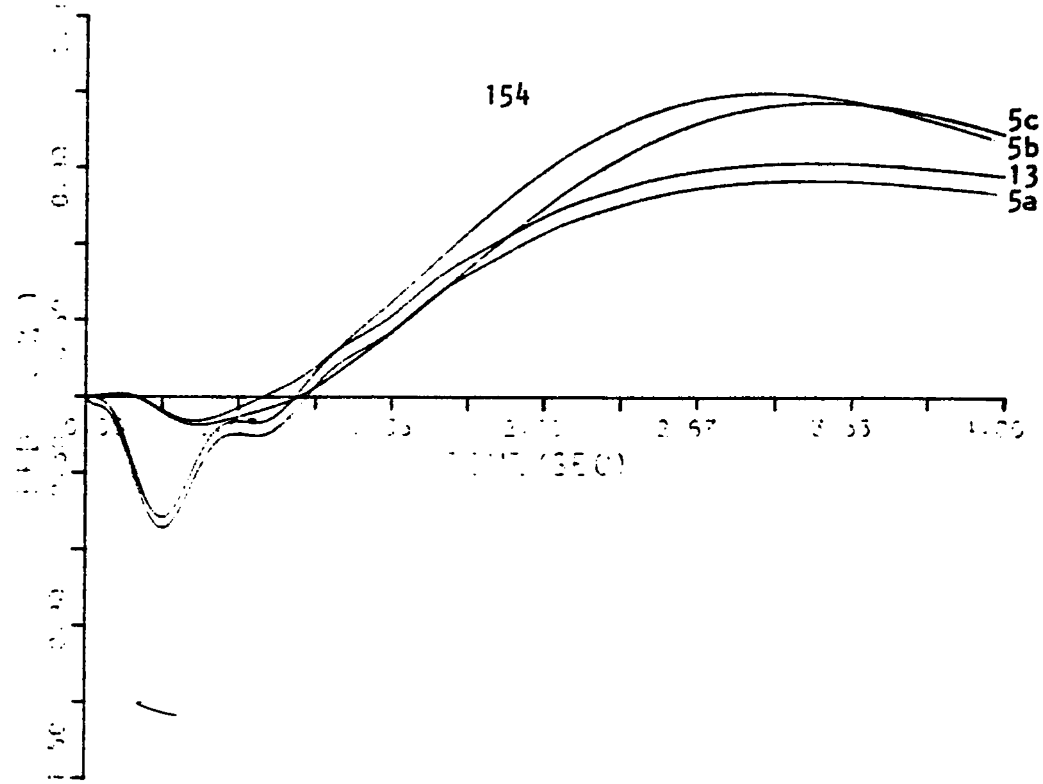


Figure 4.29. Exciter output voltage response to a 1.0% step change in P_L.

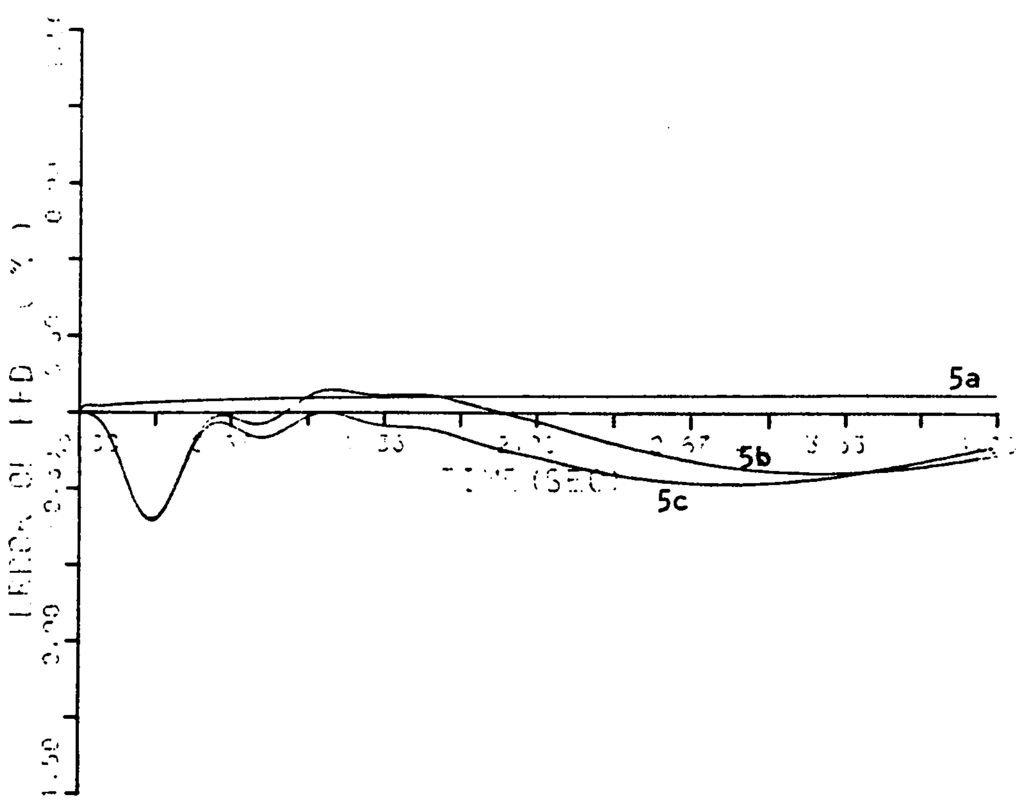


Figure 4.30. Exciter output voltage error to a 1.0% step change in P_L.

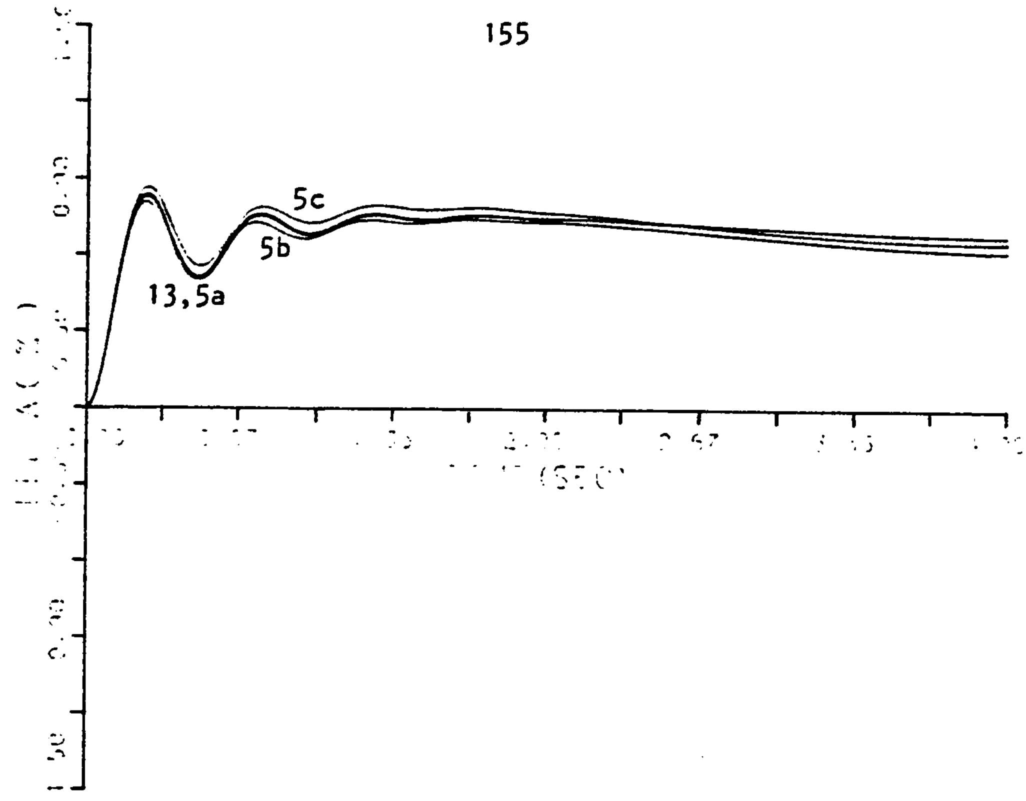


Figure 4.31. Rotor angle response to a 1.0% step change in P_L.

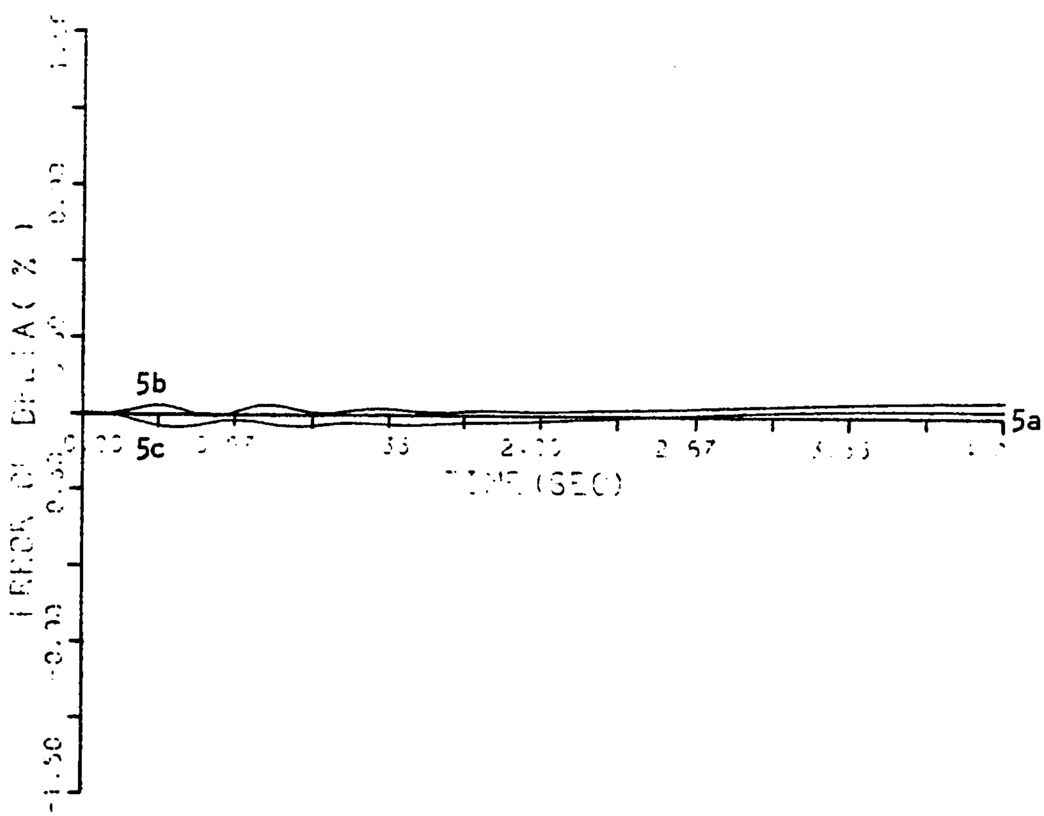


Figure 4.32. Rotor angle error to a 1.0% step change in P_L.

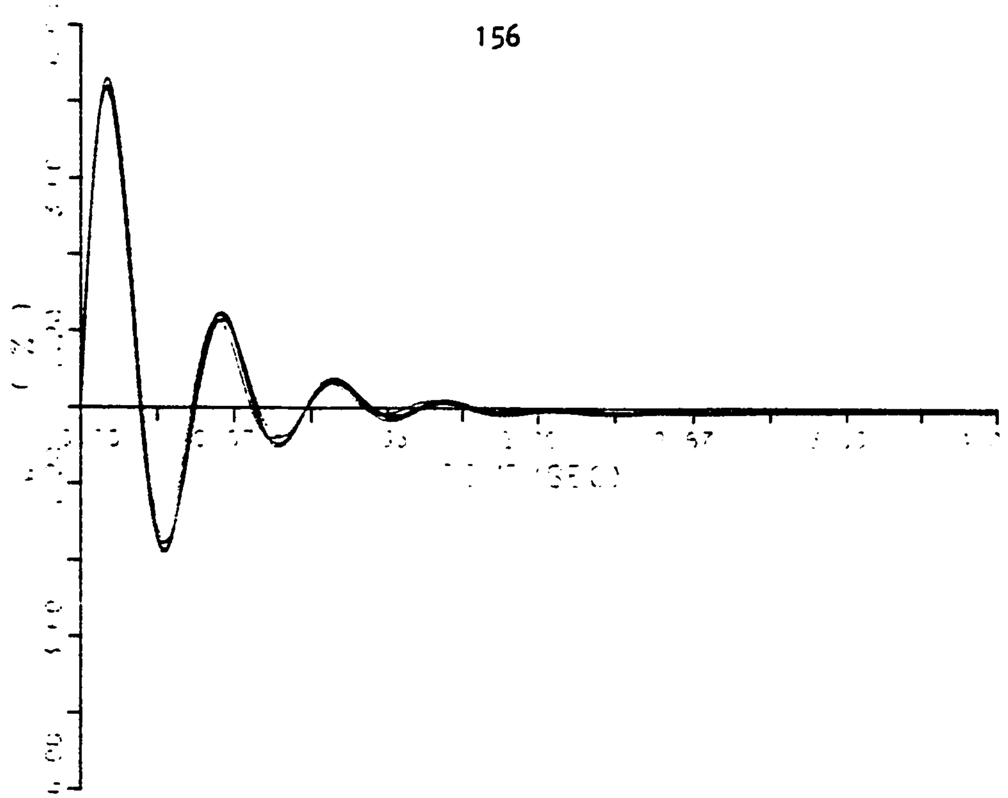


Figure 4.33. Rotor frequency response to a 1.0% step change in P_L.

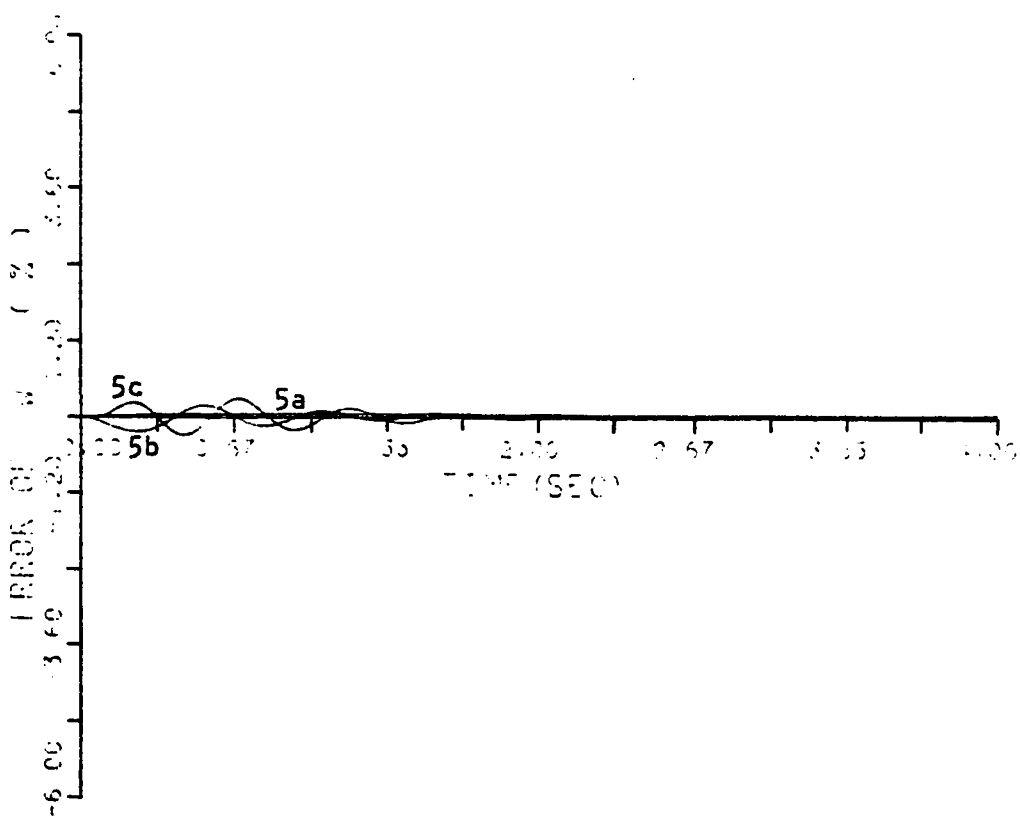


Figure 4.34. Rotor frequency error to a 1.0% step change in P_L.

Chapter 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

The input-output performance indices yield a reliable procedure for mode selection for modal equivalents, as an alternative to choosing modes according to the distance to the origin. Moreover, the error incurred in eliminating a mode or set of modes can be easily estimated in term of input-output performance indices. Also this error can be used as a criterion to check the accuracy of each reduced order model. The results obtained in Chapter 3 show that a fifth order reduced model is practically equivalent to the complete system. However, other reduced order models such as second, third and fourth order models give reasonable results for some outputs with acceptable accuracy.

The difficulty in applying the method of the eigenvalue analysis lies in the large computer storage requirement for large interconnected power systems and the numerous number of possible operating conditions.

With today computers unit large storage is available, the first problem is solved. A solution to the second problem lies in:

* Finding an "indicator" which will show the designer the conditions which will give rise to undamped oscillations without having to carry out an extensive search. This is attained by changes in the operating point from light load to heavy load conditions.

The results obtained in Chapter 4 show that using classical Engineering models are acceptable without ignoring the damping of the damper windings. This is clear from the value ΔD obtained in the compensated second order model which is close to the value obtained from Kimbark's formula. The difference between ΔD and Kimbark value is due to the field circuit which is negligible with respect to D_q (q-axis damping) as can be seen from the formula. By using the damping of the damper windings in the classical third and fourth order model a good result obtained that is more accurate than the modal equivalent. Finally, the results of the classical fifth order Engineering model are acceptable and we do not need to compensation for the parameters.

From the results obtained in Chapter 4, the following conclusion can be obtained:

* The damping of a Generating unit is affected by:

  a. All machine windings among which the q-axis amortisseuer windings give the most important effect for the important oscillation frequencies.

  b. Excitation system.

  c. Turbine & Governor system.

  d. Network and load representation.

* As a result, a detailed machine model including the damper windings, should be adopted for the low frequency oscillation calculations or at least don't go below classical third order Engineering model. Otherwise, incorrect results may be obtained in some cases especially when the oscillation frequency is not too low.

* Modification of all the parameters is not necessary to attain a good result, only the damping coefficient due to the damper windings is needed. This is clear from the third and fourth classical order models.

5.2 RECOMMENDATIONS

* The choice of the retained state variables together with the retained eigenvalues can also be done utilizing the input-output performance indices. The state variable corresponding to an eigenvalue would be the one that has the largest sensitivity ($|\frac{\partial x_i}{\partial \gamma_j}|$)

* The damping power of the damper windings can be calculated using Kimbark formula and further work can be done to modify Kimbark formula to incorporate the field winding.

* Further work can be done to extend the modal methodology of this thesis for the multimachine case and to check the results with the classical Engineering models.

* Also, comparison of the modal approach described in this thesis with some other equivalencing approaches (e.g. Equivalent to a Time-Scale in power system) should be made.

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APPENDICES

Appendix A3-I

CALCULATION OF THE OPERATING POINT OF A SYNCHRONOUS MACHINE CONNECTED TO AN INFINITE BUS

Figure 1 shows a one machine infinite bus system. Assuming that the infinite bus voltage is the reference

$$\text{power} = i^{*} v_B$$

$$i = P - jQ/v_B^{*} = I\angle\phi$$

where

- P active power received at the infinite bus
- Q reactive power received at the infinite bus
- v_B infinite bus voltage
- ϕ power factor angle

The terminal voltage v_t is

$$V_t = v_B + jx_e i$$

$$V_t = v_{t_0} \angle^{\Theta}$$

where the angle of V_t is equal to Θ

$$E_q = v_B + j(x_q + x_e)\, i$$

$$E_q = E_{q_0} \angle^{\delta}$$

where δ is the power angle.

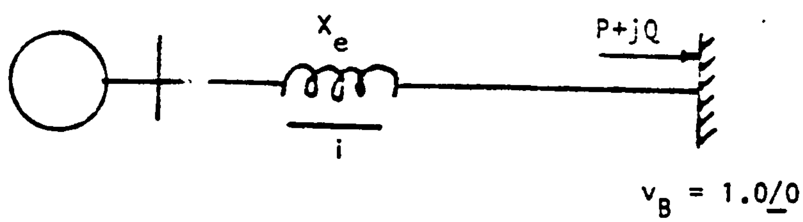


Figure 1. One machine connected to an infinite bus.

The direct and the quadrature axis currents and voltages are

$$I_{q_0} = I \cos (\phi + \delta)$$

$$I_{d_0} = I \sin (\phi + \delta)$$

$$v_{q_0} = v_B \cos (\delta)$$

$$v_{d_0} = v_B \sin (\delta)$$

$$E'_{q_0} = v_{q_0} + I_{d_0} (x'_d + x_e)$$

$$E'_{d_0} = v_{d_0} - I_{q_0} (x'_q + x_e)$$

$$V_{t_{d0}} = v_{t_0} \sin (\delta - \Theta)$$

$$V_{t_{q0}} = v_{t_0} \cos (\delta - \Theta)$$

Figure 2 shows a phasor diagram of a synchronous machine explaining the equations of the current and the voltage.

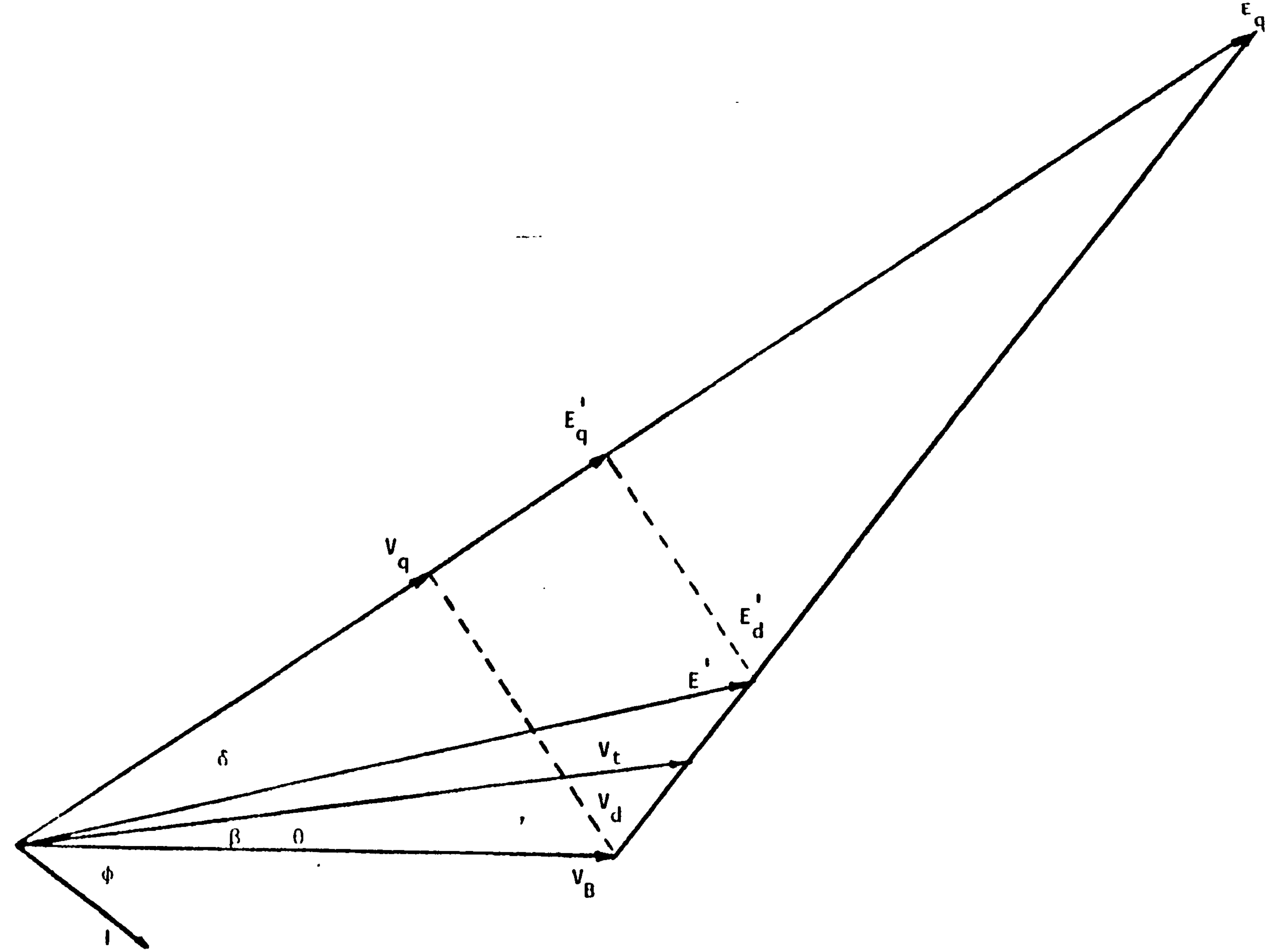


Figure 2. Phasor diagram of a synchronous machine.

Appendix A3-II

STATE VARIABLES REPRESENTATION OF A GENERATING UNIT

1. State Variables Representation of a Synchronous Machine

The synchronous machine is modeled by the d–q equivalent model. The equivalent circuits of the machine with damper windings are illustrated in Fig. 1. The following relations are assumed,

$$X_q = X_{\ell a} + X_{mq} \tag{1}$$

$$X_d = X_{\ell a} + X_{md} \tag{2}$$

$$X_{kq} = X_{\ell kq} + X_{mq} \tag{3}$$

$$X_{kd} = X_{\ell kd} + X_{md} \tag{4}$$

$$X_{fd} = X_{\ell fd} + X_{md} \tag{5}$$

The flux linkage equation can be written;

$$
\begin{bmatrix} \lambda_q \\ \lambda_d \\ \lambda_{kq} \\ \lambda_{kd} \\ \lambda_f \end{bmatrix}
=
\begin{array}{c|ccccc}
 & q & d & kq & kd & f \\ \hline
q & -X_q & & X_{mq} & & \\
d & & -X_d & & X_{md} & X_{md} \\
kq & -X_{mq} & & X_{kq} & & \\
kd & & -X_{md} & & X_{kd} & X_{md} \\
f & & -X_{md} & & X_{md} & X_f
\end{array}
\begin{bmatrix} i_q \\ i_d \\ i_{kq} \\ i_{kd} \\ i_f \end{bmatrix}
\qquad (6)
$$

The voltage equations are

$$V_q = -r_a i_q + \frac{d}{dt}\lambda_q + \omega\lambda_d \qquad (7)$$

$$V_d = -r_a i_d + \frac{d}{dt}\lambda_d - \omega\lambda_q \qquad (8)$$

$$V_{kq} = 0 = -r_{kq}\, i_{kq} + \frac{d}{dt}\lambda_{kq} \qquad (9)$$

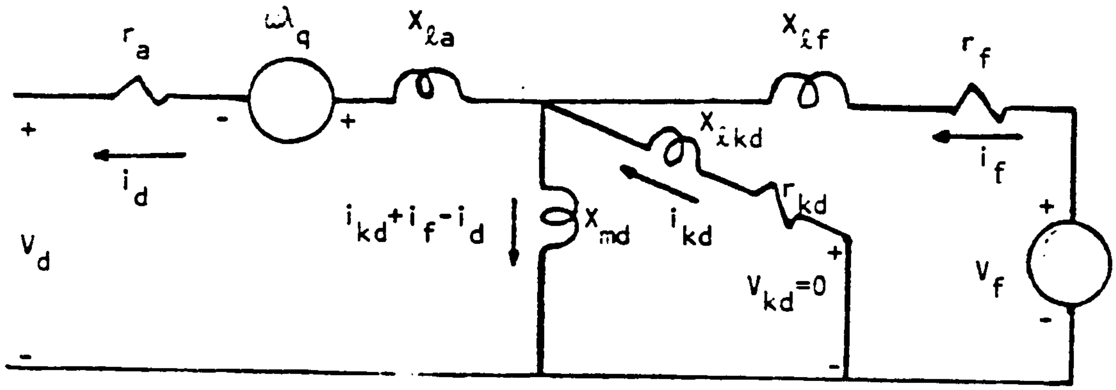
$$V_{kd} = 0 = r_{kd} i_{kd} + \frac{d}{dt}\lambda_{kd} \qquad (10)$$

$$V_f = r_f i_f + \frac{d}{dt}\lambda_f \qquad (11)$$

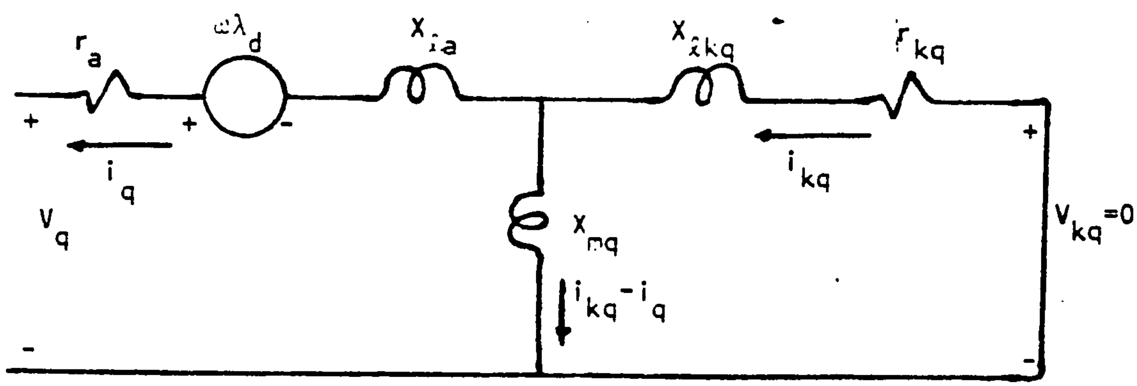
Substituting about λ's from (6), so Eqns. (7) to (11) are

$$V = Ri + PXi + \omega X_1 i \qquad (12)$$

where



d-axis equivalent circuit.



q-axis equivalent circuit.

Figure 1. Equivalent circuit of d-axis and q-axis of a synchronous generator.

$$\underline{v}^t = [V_q \;\; V_d \;\; 0 \;\; 0 \;\; V_f]$$

$$i^t = [i_q \;\; i_d \;\; i_{kq} \;\; i_{kd} \;\; i_f]$$

and $$P = \frac{d}{dt}$$

$$R = \begin{bmatrix} -r_a & & & & \\ & -r_a & & & \\ & & r_{kq} & & \\ & & & r_{kd} & \\ & & & & r_{fd} \end{bmatrix}$$

$$X = \begin{bmatrix} -X_q & & X_{mq} & & \\ & -X_d & & X_{md} & X_{md} \\ -X_{mq} & & X_{kq} & & \\ & -X_{md} & & X_{kd} & X_{md} \\ & -X_{md} & & X_{md} & X_{fd} \end{bmatrix}$$

$$X_1 = \begin{array}{|c|c|c|c|c|} \hline & -X_d & & X_{md} & X_{md} \\ \hline X_q & & -X_{mq} & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline \end{array}$$

The dimension of Eqn. (12) is volt. To normalize it, divide all the terms by the base voltage V_B as an example take the V_q itself.

$$\frac{V_q}{V_B} = \frac{-r_a}{V_B} i_q \frac{I_B}{I_B} + \frac{d}{dt}\left(- \frac{X_q i_q}{V_B} \frac{I_B \Omega_B}{I_B \Omega_B} + \frac{X_{mq} i_{kq}}{V_B} \frac{I_B \Omega_B}{I_B \Omega_B}\right) + \omega \left(- \frac{X_d i_d}{V_B} \frac{I_B \Omega_B}{I_B \Omega_B}\right.$$

$$\left. + \frac{X_{md}}{V_B} i_{kd} \frac{I_B \Omega_B}{I_B \Omega_B} + \frac{X_{md} i_f}{V_B I_B \Omega_B} I_B \Omega_B \right) \qquad (13)$$

where

V_B base voltage in Volt

I_B base current in Ampere.

Equation (13) in p.u. is

$$V_q = -r_a i_q + \frac{1}{\Omega_B}\frac{d}{dt}(-X_q i_q + X_{mq} i_{kq}) + \frac{\omega}{\Omega_B}(-X_d i_d + X_{md} i_{kd} + X_{md} i_f) \tag{14}$$

Equation (12) in matrix form can be written as

$$v = Ri + \frac{1}{\Omega_B} PXi + \frac{1}{\Omega_B} X_1 i \tag{15}$$

All terms of Eqn. (15) are in p.u. except t, ω and Ω_B are in second and radian/sec respectively. The field voltage V_f corresponds (at steady state) to a field current $\frac{V_f}{r_f}$. This in turn corresponds to a peak stator EMF ($\frac{V_f}{r_f} X_{md}$).

So usually the d axis stator EMF corresponds to a field voltage V_f can be represented by

$$E_{FD} = \frac{V_f}{r_f} X_{md} \quad \text{p.u.} \tag{16}$$

Equation (16) modifies the 5th row in Eqn. (15) using E_{FD} instead of V_f. Equation (15) are nonlinear, so the method of linearization around certain operating condition is used.

That is to say, the variables v, i and ω are allowed to perturb by an amount Δv, Δi and $\Delta\omega$ from a certain steady state operating condition V, I and Ω, where Ω steady state speed and

$$v = V + \Delta v$$
$$i = I + \Delta i \qquad (17)$$
$$\omega = \Omega + \Delta\omega$$

At steady state operating condition assumed that

1. $I_{kd} = I_{kq} = 0$ (initial condition of the damper windings current are neglected).

2. Ω_B = rated (synchronous speed) rad/sec.

3. The system is balanced, so zero sequence is neglected ($v_o=i_o=0$).

Linearization of Eqn. (15) yields to with droping the variable Δ for convenience

$$v = R_1 X_w + \frac{1}{\Omega_B} X_2 P X_w + \frac{1}{\Omega_B} X_3 \omega \qquad (18)$$

where

$$V^t = [\Delta V_q \;\; \Delta V_d \;\; 0 \;\; 0 \;\; \Delta E_{FD}]$$

$$X_w = [\Delta i_q \; \Delta i_d \; \Delta i_{kq} \; \Delta i_{kd} \; \Delta i_f]$$

$$\omega^t = [\Delta\omega]$$

and

$$X_2 = \begin{bmatrix} -X_q & & X_{mq} & & \\ & -X_d & & X_{md} & X_{md} \\ -X_{mq} & & X_{kq} & & \\ & -X_{md} & & X_{kd} & X_{md} \\ & \dfrac{-X_{md}^2}{r_f} & & \dfrac{X_{md}^2}{r_f} & X_{fd}\dfrac{X_{md}}{r_f} \end{bmatrix}$$

$$R_1 = \begin{bmatrix} -r_a & -X_d & & X_{md} & X_{md} \\ X_q & -r_a & -X_{mq} & & \\ & & r_{kq} & & \\ & & & r_{kd} & \\ & & & & X_{md} \end{bmatrix}$$

$$X_3 = \begin{bmatrix} \psi_{do} \\ \psi_{qo} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where

$$\psi_{do} = -X_d I_{do} + X_{md} I_{fo}$$

$$\psi_{qo} = X_q I_{qo}$$

2. <u>State Variables Representations of a Shaft and Governor Turbine</u>

The principal control systems directly affect a synchronous generator: The boiler control, governor and the exciter. First of all we shall take the shaft and governor/turbine. The nonreheat steam unit governor turbine type is used in this study. The transfer function block diagram for a nonreheat steam turbine and governor is shown in Fig. 2 in which a central power input signal is also shown.

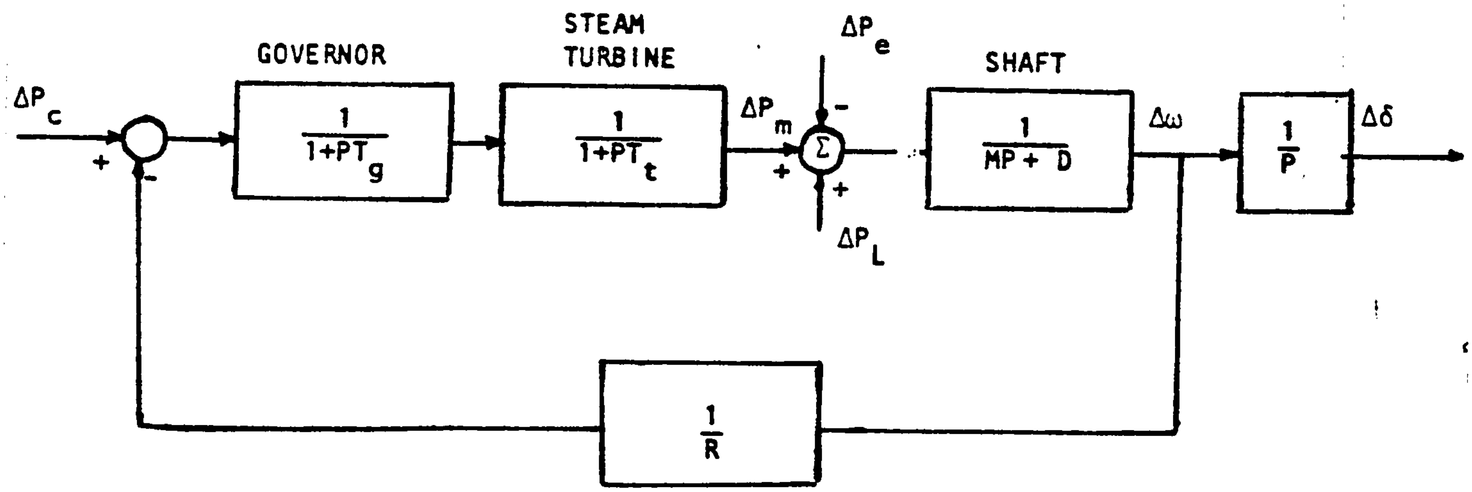


Figure 2. Transfer function block diagram for a nonreheat steam turbine with the shaft.

The parameters shown in Fig. 2 are defined as follows:

R	Steady state speed regulation (sec^{-1})
T_g	Governor time constant (sec)
T_s	Steam turbine time constant (sec)
D	Damping coefficient (sec)
M	inertia constant (sec^2).

The differential equations are:

$$P\Delta\delta = \Delta\omega \tag{19}$$

$$P\Delta\omega = -\frac{D}{M}\Delta\omega + \frac{1}{M}\Delta P_m - \frac{1}{M}\Delta P_e - \frac{1}{M}\Delta P_L \tag{20}$$

The electrical power ΔP_e can be calculated from

$$P_e = i_q\lambda_d - i_d\lambda_q \tag{21}$$

Substituting λ_d and λ_q in Eqn. (21) and differentiating yielding:

$$\Delta P_e = [\, E_1 \quad E_2 \quad E_3 \quad E_4 \quad E_5 \,] \begin{pmatrix} \Delta i_q \\ \Delta i_d \\ \Delta i_q \\ \Delta i_{kd} \\ \Delta i_f \end{pmatrix} \qquad (22)$$

where

$$E_1 = (X_q - X_d)\, I_{do} + X_{md}\, I_{fo}$$
$$E_2 = (X_q - X_d)\, I_{qo}$$
$$E_3 = -X_{mq}\, I_{do}$$
$$E_4 = X_{md}\, I_{qo}$$
$$E_5 = X_{md}\, I_{qo}$$

Equation (22) incorporates the interaction between the dynamics of the electrical machine windings and that of the rotating masses.

So, Eqn. (20)

$$P\Delta\omega = -\frac{D}{M}\Delta\omega + \frac{1}{M}\Delta P_m - \frac{1}{M}\begin{bmatrix} E_1 & E_2 & E_3 & E_4 & E_5 \end{bmatrix}\begin{bmatrix} \Delta i_q \\ \Delta i_d \\ \Delta i_{kq} \\ \Delta i_{kd} \\ \Delta i_f \end{bmatrix} + \frac{1}{M}\Delta P_L \tag{23}$$

$$P\Delta P_m = \frac{1}{T_t}\Delta P_m - \frac{1}{T_t}\Delta P_g \tag{24}$$

$$P\Delta P_g = -\frac{1}{RT_g}\Delta\omega - \frac{1}{T_g}\Delta P_g + \frac{1}{T_g}\Delta P_c \tag{25}$$

The differential Equations describe the shaft ($\Delta\delta$, $\Delta\omega$) shows that the input to the rotating masses are the machine voltage/or currents. Hence, these equations have to be coupled with the machine winding equations and network in order to reflect the electromechanical behavior of the synchronous machine under study.

The differential equations can be combined in a compact form as

$$p\begin{bmatrix}\Delta\delta\\ \Delta\omega\\ \Delta P_m\\ \Delta P_g\end{bmatrix} = \begin{bmatrix} & 1 & & \\ & \frac{D}{M} & \frac{1}{M} & \\ & & -\frac{1}{T_t} & \frac{1}{T_t}\\ & -\frac{1}{RT_g} & & -\frac{1}{T_g}\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\omega\\ \Delta P_m\\ \Delta P_g\end{bmatrix} - \begin{bmatrix} & & & & \\ \frac{E_1}{M} & \frac{E_2}{M} & \frac{E_3}{m} & \frac{E_4}{M} & \frac{E_5}{M}\\ & & & & \\ & & & & \end{bmatrix}\begin{bmatrix}\Delta i_q\\ \Delta i_d\\ \Delta i_{kq}\\ \Delta i_{kd}\\ \Delta i_f\end{bmatrix} + \begin{bmatrix} \\ \frac{1}{M}\\ \\ \\ \end{bmatrix}\Delta P_L + \begin{bmatrix} \\ \\ \\ \frac{1}{T_g}\end{bmatrix}\Delta P_C \qquad (26)$$

In state space form as

$$PX_m = A_m X_m + A_{m_1} X_w + B_{m_1} \Delta P_L + B_{m_2} \Delta P_c \tag{27}$$

where

$$A_m = \begin{bmatrix} & 1 & & \\ & -\frac{D}{M} & \frac{1}{M} & \\ & & -\frac{1}{T_t} & \frac{1}{T_t} \\ & -\frac{1}{RT_g} & & -\frac{1}{T_g} \end{bmatrix} \qquad B_{m_1} = \begin{bmatrix} \\ \frac{1}{M} \\ \\ \\ \end{bmatrix}$$

$$A_{m_1} = \begin{bmatrix} & & & & \\ -\frac{E_1}{M} & -\frac{E_2}{M} & -\frac{E_3}{M} & -\frac{E_4}{M} & -\frac{E_5}{M} \\ & & & & \\ & & & & \end{bmatrix} \qquad B_{m_2} = \begin{bmatrix} \\ \\ \\ \frac{1}{T_g} \end{bmatrix}$$

3. State Variables Representation of a Automatic Voltage Regulator and Exciter

The primary function of excitation system is to maintain a constant terminal voltage, the damping of rotor oscillations by supplying a speed signal to the excitation system is becoming increasingly important, since most of the synchronous generator units are equipped with the IEEE type 1 excitation systems, they will be used in our study.

The transfer function block diagram of a type 1 excitation system is given in Fig. 3 in which a control input signal is also shown.

T is a transformation matrix transforms the machine qd voltage components to its terminal voltage magnitude ΔV_t. Hence,

$$V_t^2 = V_t \cdot V_t = (V_{to} + \Delta V_t)(V_{to} + \Delta V_t)$$
$$= V_{to}^2 + 2V_{to}\Delta V_t \qquad (28)$$

So

$$V_t^2 = V_{t_q}^2 + V_{t_d}^2$$

or

$$2V_{to}\Delta V_t = 2V_{t_{qo}}\Delta V_q + 2V_{t_{do}}\Delta V_d$$

$$\Delta V_t = T\,\Delta V_{t_{q,d}} \qquad (29)$$

where

$$T = \begin{pmatrix} \frac{V_{t_{qo}}}{V_{to}} & \frac{V_{t_{do}}}{V_{to}} \end{pmatrix}$$

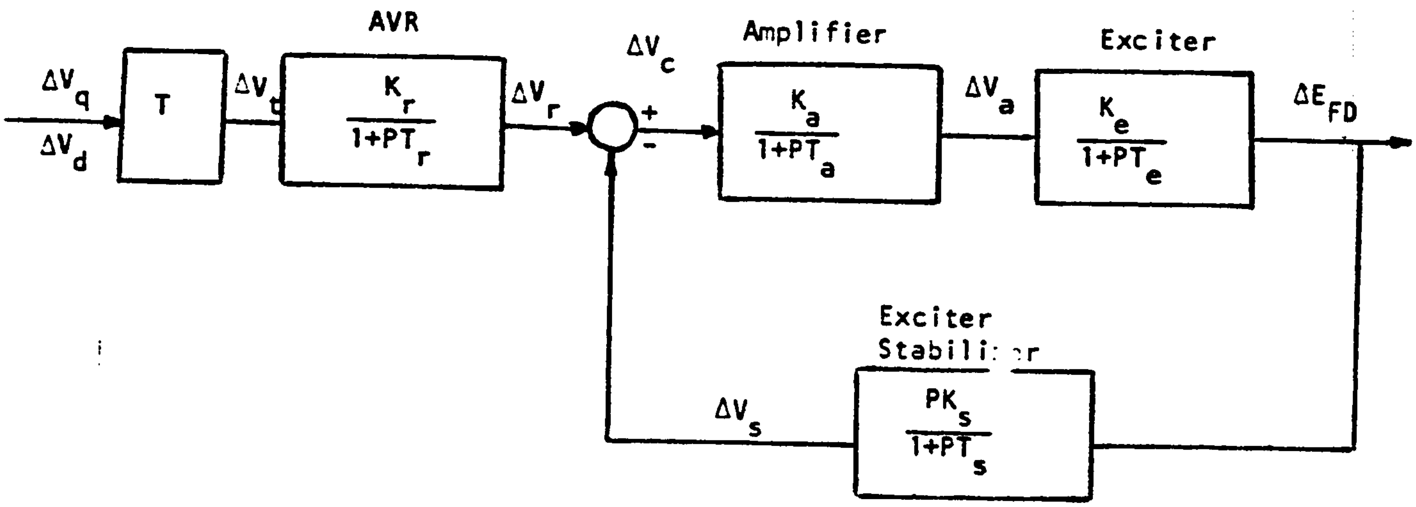


Figure 3. Transfer function block diagram for an IEEE Type 1 excitation system showing the AVR Exciter and Exciter stabilizer blocks.

The parameters given in Fig. 3 are defined as follows:

K_r AVR gain

T_r AVR time constant sec.

K_a Amplifier gain

T_a Amplifier time constant sec.

K_e Exciter gain

T_e Exciter time constant sec.

K_s Exciter stabilizer gain

T_s Exciter stabilizer time constant sec.

The differential Equation describe the model are

$$P\Delta E_{FD} = -\frac{1}{T_e}\Delta E_{FD} + \frac{K_e}{T_e}\Delta V_a \tag{30}$$

$$P\Delta v_a = -\frac{1}{T_a}\Delta V_a - \frac{K_a}{T_a}\Delta V_r - \frac{K_a}{T_a}\Delta V_s + \frac{K_a}{T_a}\Delta V_c \tag{31}$$

$$P\Delta V_r = -\frac{1}{T_r}\Delta V_r + \frac{K_r V_{tqo}}{T_r V_{to}}\Delta V_q + \frac{K_r V_{tdo}}{T_r V_{to}}\Delta V_d \tag{32}$$

$$P\Delta V_s = -\frac{K_s}{T_s T_e}\Delta E_{FD} + \frac{K_s K_e}{T_s T_e}\Delta V_a - \frac{1}{T_s}\Delta V_s \tag{33}$$

or in a matrix form

$$PX_e = A_e X_e + B_{e1}\, u + B_{e2}\Delta V_c \tag{34}$$

where

$$X_e^t = [\Delta E_{FD} \quad \Delta V_a \quad \Delta V_r \quad \Delta V_s]$$
$$u^t = [\Delta V_q \quad \Delta V_d]$$

$$A_e = \begin{array}{|c|c|c|c|} \hline -\frac{1}{T_e} & \frac{K_e}{T_e} & & \\ \hline & -\frac{1}{T_a} & -\frac{K_a}{T_a} & -\frac{K_a}{T_a} \\ \hline & & -\frac{1}{T_r} & \\ \hline -\frac{K_s}{T_s T_e} & \frac{K_s K_e}{T_s T_e} & & -\frac{1}{T_s} \\ \hline \end{array}$$

$$B_{e_1} = \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \frac{K_r V t_{qo}}{V_{t_o} T_r} & \frac{K_r V t_{do}}{V_{to} T_r} \\ \hline & \\ \hline \end{array} \qquad B_{e_2} = \begin{array}{|c|} \hline \\ \hline \frac{K_a}{T_a} \\ \hline \\ \hline \\ \hline \end{array}$$

Appendix A3-III

COMPLETE GENERATING UNIT REPRESENTATION INCLUDING GOVERNOR, TURBINE, AVR AND EXCITER

The differential equations derived in Appendix A3-I representing the winding of Eqn. (18) may be coupled with those representing rotating masses (shaft), the governor and turbine of Eqn. (26) and those representing the AVR and Exciter of Eqns. (30), resulting in a model representing one complete generating unit.

The complete generating unit of Fig. 3.2 may be expressed by the state space model:

$$MX + \frac{1}{\Omega_B} KPX = B_1 u + B_2 W \qquad (1)$$

where

$$X^t = [\Delta i_q \; \Delta i_d \; \Delta i_{kq} \; \Delta i_{kd} \; \Delta i_f \; \Delta\delta \; \Delta\omega \; \Delta E_{FD} \; \Delta V_\alpha \; \Delta V_r \; \Delta V_s \; \Delta P_m \; \Delta P_g]$$

$$u^t = [\Delta V_q \;\; \Delta V_d]$$

and $\quad W^t = \text{control input} = [\Delta P_L \;\; \Delta V_c \;\; \Delta P_c]$

The matrices M, K, B_1 and B_2 are defined as follows:

$$
M = \begin{bmatrix}
-r & -X_d & & X_{md} & X_{md} & & \frac{\psi_{do}}{\omega_B} & & & & & & \\
X_q & -r & -X_{mq} & & & & \frac{\psi_{qo}}{\omega_B} & & & & & & \\
 & & r_{kq} & & & & & & & & & & \\
 & & & r_{kd} & & & & & & & & & \\
 & & & & X_{md} & & & & & -1 & & & \\
 & & & & & & 1 & & & & & & \\
-\frac{E_1}{M} & -\frac{E_2}{M} & -\frac{E_3}{M} & -\frac{E_4}{M} & -\frac{E_5}{M} & & -\frac{D}{M} & & & & & \frac{1}{M} & \\
 & & & & & & & -\frac{1}{T_e} & \frac{K_e}{T_e} & & & & \\
 & & & & & & & & -\frac{1}{T_a} & -\frac{K_a}{T_a} & -\frac{K_a}{T_a} & & \\
 & & & & & & & & & -\frac{1}{T_r} & & & \\
 & & & & & & & -\frac{K_s}{T_s T_e} & \frac{K_s K_e}{T_s T_e} & & -\frac{1}{T_s} & & \\
 & & & & & & & & & & & -\frac{1}{T_t} & \frac{1}{T_t} \\
 & & & & & & -\frac{1}{RT_g} & & & & & & -\frac{1}{T_g}
\end{bmatrix}
$$

38

$$K = \begin{bmatrix}
-X_q & & X_{mq} & & & & & & & & & & \\
& -X_d & & X_{md} & X_{md} & & & & & & & & \\
-X_{mq} & & X_{kq} & & & & & & & & & & \\
& -X_{md} & & X_{kd} & X_{md} & & & & & & & & \\
& -\frac{X_{md}^2}{r_f} & & \frac{X_{md}^2}{r_f} & \frac{X_{md}X_{fd}}{r_f} & & & & & & & & \\
& & & & & -\Omega_B & & & & & & & \\
& & & & & & -\Omega_B & & & & & & \\
& & & & & & & -\Omega_B & & & & & \\
& & & & & & & & -\Omega_B & & & & \\
& & & & & & & & & -\Omega_B & & & \\
& & & & & & & & & & -\Omega_B & & \\
& & & & & & & & & & & -\Omega_B & \\
& & & & & & & & & & & & -\Omega_B
\end{bmatrix}$$

$$
B_1 = \begin{bmatrix}
1 & \\
 & 1 \\
 & \\
 & \\
 & \\
 & \\
 & \\
 & \\
 & \\
-\dfrac{K_r V_{tqo}}{V_{to} T_r} & -\dfrac{K_r V_{tdo}}{V_{to} T_r} \\
 & \\
 & \\
 & \\
\end{bmatrix}
$$

$$B_2 = \begin{bmatrix} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ -\frac{1}{M} & & \\ & & \\ & -\frac{K_a}{T_a} & \\ & & \\ & & \\ & & \\ & & -\frac{1}{T_g} \end{bmatrix}$$

DYNAMIC STABILITY ANALYSIS OF ONE MACHINE INFINITE BUS SYSTEM

The one machine infinite bus system consists of a synchronous generator connected to an infinite bus through a transmission line. It is assumed that the voltage and frequency of the infinite bus are not altered due to real and reactive power flow. Figure 3.2 shows a generating unit connected to infinite bus bar through a T.L. We can express the quadrature and direct axis voltage in term of line parameters and state variables. So, ΔV_q and ΔV_d in matrix form are defined as follows:

$$\begin{bmatrix} \Delta V_q \\ \Delta V_d \end{bmatrix} = \begin{bmatrix} r_L & X_L & 0 & 0 & 0 & -V_B \sin\delta_o & \dfrac{X_L i_{do}}{\Omega_B} \\ -X_L & r_L & 0 & 0 & 0 & V_B \cos\delta_o & -\dfrac{X_L i_{qo}}{\Omega_B} \end{bmatrix} \begin{bmatrix} \Delta i_q \\ \Delta i_d \\ \Delta i_{kq} \\ \Delta i_{kd} \\ \Delta i_f \\ \Delta\delta \\ \Delta\omega \end{bmatrix} + P \begin{bmatrix} \dfrac{X_L}{\Omega_B} & \\ & \dfrac{X_L}{\Omega_B} \end{bmatrix} \begin{bmatrix} \Delta i_q \\ \Delta i_d \end{bmatrix} \tag{2}$$

So, substituting 2 in 1, yields

$$M_1 X + \frac{1}{\Omega_B} K_1 PX = B_2 W \qquad (3)$$

or in compact form

$$PX = AX + B_4 W \qquad (4)$$

where

$$A = -\Omega_B K_1^{-1} M_1$$
$$B_4 = \Omega_B K_1^{-1} B_2$$

and

$$
M_1 = \begin{bmatrix}
-\hat{r} & -\hat{X}_d & & X_{md} & X_{md} & V_B \sin\delta_o & \frac{\hat{\psi}_{do}}{\Omega_B} & & & & & & \\
\hat{X}_q & -\hat{r} & -X_{mq} & & & -V_B \cos\delta_o & \frac{\hat{\psi}_{qo}}{\Omega_B} & & & & & & \\
 & & r_{kq} & & & & & & & & & & \\
 & & & r_{kd} & & & & & & & & & \\
 & & & & X_{md} & & & & & -1 & & & \\
 & & & & & & 1 & & & & & & \\
-\frac{E_1}{M} & -\frac{E_2}{M} & -\frac{E_3}{M} & -\frac{E_4}{M} & -\frac{E_5}{M} & & -\frac{D}{M} & & & & & \frac{1}{M} & \\
 & & & & & & & -\frac{1}{T_e} & \frac{K_e}{T_e} & & & & \\
 & & & & & & & & -\frac{1}{T_a} & -\frac{K_a}{T_a} & -\frac{K_a}{T_a} & & \\
M_{10,1} & M_{10,2} & & & & M_{10,6} & M_{10,7} & & & -\frac{1}{T_r} & & & \\
 & & & & & & & -\frac{K_s}{T_s T_e} & \frac{K_s K_e}{T_s T_e} & & -\frac{1}{T_s} & & \\
 & & & & & & & & & & & -\frac{1}{T_t} & \frac{1}{T_t} \\
 & & & & & -\frac{1}{RT_g} & & & & & & & -\frac{1}{T_g}
\end{bmatrix}
$$

where

$$\hat{r} = r_a + R_L$$

$$\hat{X}_d = X_d + X_L$$

$$\hat{X}_q = X_q + X_L$$

$$M_{10,1} = \frac{1}{T_r V_{to}} (K_r \; V_{t_{qo}} r_L - K_r V_{t_{do}} X_L)$$

$$M_{10,2} = \frac{1}{T_r V_{to}} (K_r V_{t_{qo}} X_L + K_r V_{t_{do}} r_L)$$

$$M_{10,6} = \frac{1}{T_r V_{to}} (-K_r V_{t_{qo}} V_B \sin \delta_o + K_r V_{t_{do}} V_B \cos \delta_o)$$

$$M_{10,7} = \frac{1}{\Omega_B T_r V_{to}} (K_r I_{do} V_{t_{qo}} X_L - K_r I_{qo} V_{t_{do}} X_L)$$

$$
K_1 = \begin{bmatrix}
-X_q & & X_{mq} & & & & & & & & & & \\
& -X_d & & X_{md} & X_{md} & & & & & & & & \\
-X_{mq} & & X_{kq} & & & & & & & & & & \\
& -X_{md} & & X_{kd} & X_{md} & & & & & & & & \\
& -\frac{X_{md}^2}{r_f} & & \frac{X_{md}^2}{r_f} & \frac{X_{md}X_{fd}}{r_f} & & & & & & & & \\
& & & & & -\Omega_B & & & & & & & \\
& & & & & & -\Omega_B & & & & & & \\
& & & & & & & -\Omega_B & & & & & \\
& & & & & & & & -\Omega_B & & & & \\
\frac{X_L V_{tqo} K_r}{V_{to} T_r} & \frac{X_L V_{tdo} K_r}{V_{to} T_r} & & & & & & & & -\Omega_B & & & \\
& & & & & & & & & & -\Omega_B & & \\
& & & & & & & & & & & -\Omega_B & \\
& & & & & & & & & & & & -\Omega_B
\end{bmatrix}
$$

$$B_4 = \begin{array}{|c|c|c|}
\hline
 & & \\ \hline
 & & \\ \hline
 & & \\ \hline
 & & \\ \hline
 & & \\ \hline
 & & \\ \hline
\frac{1}{M} & & \\ \hline
 & & \\ \hline
 & \frac{K_a}{T_a} & \\ \hline
 & & \\ \hline
 & & \\ \hline
 & & \\ \hline
 & & \frac{1}{T_g} \\ \hline
\end{array}$$

Appendix A4-I

STATE VARIABLES REPRESENTATION OF A GENERATING UNIT USING CLASSICAL SECOND ORDER ENGINEERING MODEL

The differential equations describe this model are

$$\dot{\delta} = \omega - 1 \tag{1}$$

$$\dot{\omega} = -\frac{D}{M}\,\omega + \frac{1}{M}\,P_a \tag{2}$$

where

δ rotor angle rad.

ω rotor frequency rad/sec.

D Damping

M Inertia sec^2

P_a accelerating power

The accelerating power P_a is

$$P_a = P_m - P_e + P_L \tag{3}$$

where

P_e electrical power

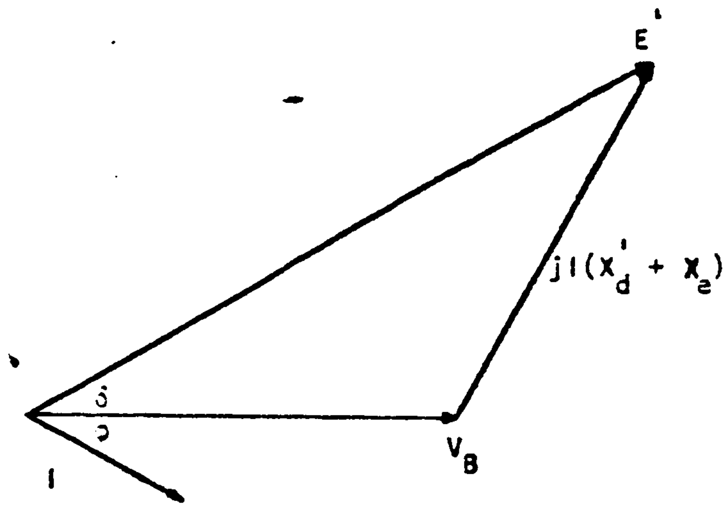


Figure 1. Phas · diagram of a voltage behind transient reactance mod· 1.

P_L load power

$$P_e = \frac{|V_B E'|}{X} \sin \delta \tag{4}$$

V_B and E' are the magnitude of the voltage of the infinite bus and the machine internal voltage.

E' can be calculated from the phasor diagram of Fig. 1

$$E' = V_B + j\,XI$$

and the reactance X is the sum of the transmission line reactance and the direct axis transient reactance.

Equations (1) and (2) after linearization are

$$\Delta\dot{\delta} = \Delta\omega \tag{5}$$

$$\Delta\dot{\omega} = -\frac{D}{M}\Delta\omega - \frac{V_B E'}{XM} \cos \delta_o \, \Delta\delta + \frac{1}{M}\Delta P_L \tag{6}$$

Considering that P_m is constant and the only input is the change in the load power.

Equations (5) and (6) in matrix form are

$$\begin{bmatrix} \dot{\Delta\delta} \\ \\ \dot{\Delta\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \\ -\frac{|V_B E'|}{XM}\cos\delta' & -\frac{D}{M} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} 0 \\ \\ \frac{1}{M} \end{bmatrix} \Delta P_L$$

or equal

$$\dot{X}_2 = A_2 X_2 + B_2 u \tag{7}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ \\ -\frac{K}{M} & -\frac{D}{M} \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 \\ \\ \frac{1}{M} \end{bmatrix}$$

$$X_2^t = [\Delta\delta \quad \Delta\omega]$$

K = is the synchronous power coefficient $= \frac{|V_B E'|}{X} \cos\delta$

Appendix A4-II

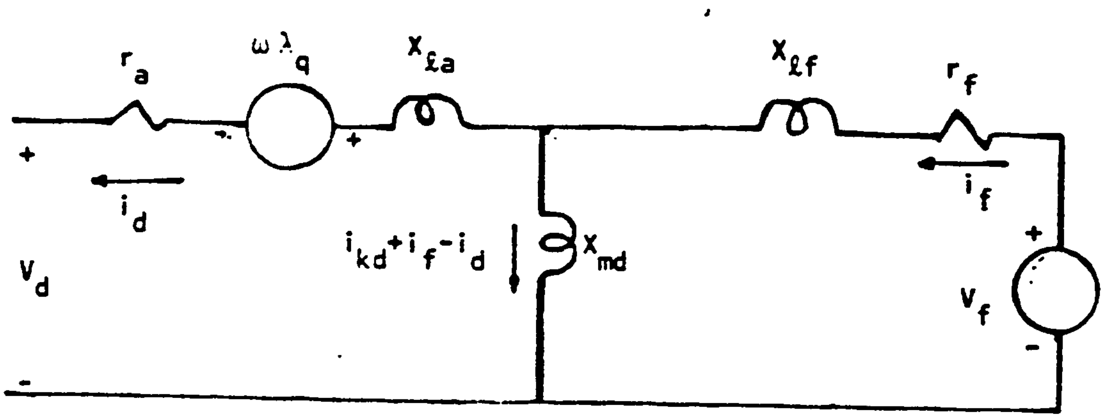
STATE VARIABLES REPRESENTATION OF THE MACHINE WINDINGS FOR CLASSICAL ENGINEERING MODEL

Assumption That Reduces a High Order to a Low Order:

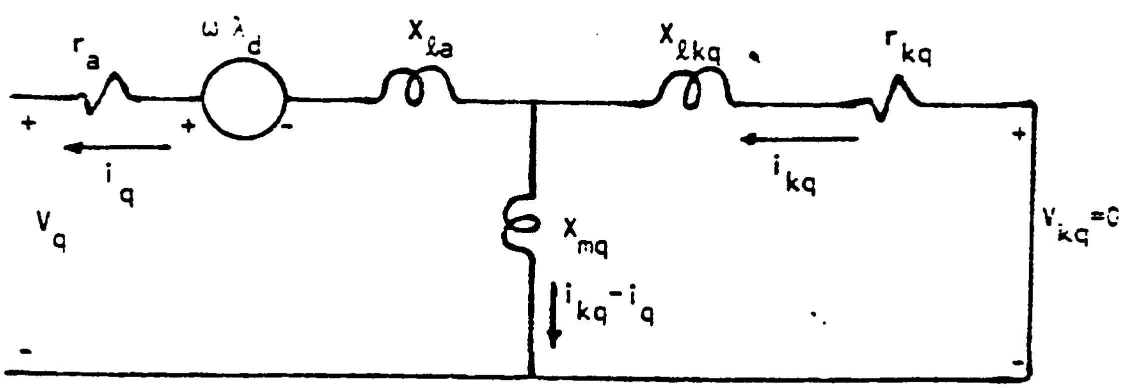
1. Stator side include two circuits q and d.
2. Rotor side includes one circuit in the d axis represented by field circuit.
3. Rotor side includes one circuit at the q axis represented by single damper circuit.
4. The transformer voltage $\frac{d\lambda_q}{dt} = \frac{d\lambda_d}{dt} = 0$

Assumption (4) reduces the two stator voltage equations to algebraic equations, while assumption (2) and (3) representing the field and the rotor by its time constant. According to these assumption the electrical part (synchronous machine, field) is represented by a second order, if assumption (3) is dropped, the electrical part is represented by a first order which is the field winding only.

We shall consider all the assumption stated above in our derivation yielding a second order. If a first order is required the reader should drop the unwanted equation.



d-axis equivalent circuit.



q-axis equivalent circuit.

Figure 1. Equivalent circuit of d-axis and q-axis of a synchronous generator.

The flux linkage equations

$$\lambda_d = \lambda_{md} - X_{\ell a} i_d \qquad (1)$$

$$\lambda_q = \lambda_{mq} - X_{\ell a} i_q \qquad (2)$$

$$\lambda_{kq} = \lambda_{mq} + X_{\ell kq} i_{kq} \qquad (3)$$

$$\lambda_F = \lambda_{md} + X_{\ell fd} i_f \qquad (4)$$

$$\lambda_{md} = X_{md} (i_f - i_d) \qquad (5)$$

$$\lambda_{mq} = X_{mq} (i_{kq} - i_q) \qquad (6)$$

The corresponding voltage equations are

$$V_d = -ri_d - \omega\lambda_q + \dot{\lambda}_d \qquad (7)$$

Since $\dot{\lambda}_d = 0$ then

$$V_d = -ri_d - \omega\dot{\lambda}_q \qquad (8)$$

$$V_q = -ri_q + \omega\lambda_d + \dot{\lambda}_q \qquad (9)$$

Also $\dot{\lambda}_q = 0$ then

$$V_q = -ri_q + \omega\lambda_d \qquad (10)$$

$$V_f = +r_f i_f + \dot{\lambda}_f \tag{11}$$

$$0 = V_{kq} = r_{kq} i_{kq} + \dot{\lambda}_{kq} \tag{12}$$

Eliminate the voltage $\omega\lambda_q$ and $\omega\lambda_d$ using Eqns. (1) - (6) and

$$E_q' = \frac{\lambda_F X_{md}}{X_f} \tag{13}$$

$$\frac{X_{md}^2}{X_f} = X_d - X_d' \tag{14}$$

$$E_d' = \frac{\lambda_{kq} X_{mq}}{X_{kq}} \tag{15}$$

$$\frac{X_{mq}^2}{X_{kq}} = X_q - X_q' \tag{16}$$

then

$$V_q = -ri_q - X_d' i_d + E_q' \tag{17}$$

$$V_d = -ri_d + X_q' i_q + E_d' \tag{18}$$

Equations (17) and (18) represent the stator voltage equation which is algebraic according to assumption (4).

Equations (11) and (12) include the derivative of the flux linkage and using:

$$E_{FD} = V_f \frac{X_{md}}{r_f} \tag{19}$$

$$T'_{do} = \frac{X_f}{r_f} \tag{20}$$

$$T'_{qo} = \frac{X_{kq}}{r_{kq}} \tag{21}$$

where

T'_{do} and T'_{qo} are the d-axis and q-axis transient open circuit time constant respectively.

Equations (11) and (12) become

$$\dot{E}'_q = \frac{E_{FD}}{T'_{do}} - \frac{E'_q}{T'_{do}} - \frac{(X_d - X'_d)}{T'_{do}} i_d \tag{22}$$

$$\dot{E}'_d = - \frac{E'_d}{T'_{qo}} + \frac{X_q - X'_q}{T'_{qo}} i_q \tag{23}$$

Equations (22) and (23) represent the second order for the electrical parts. If a first order is required, the single damper circuit is removed from the q-axis equivalent circuit and corresponding to this:

$$V_q = -ri_q - X'_d i_d + E'_q \tag{24}$$

$$V_d = -ri_d + X_q i_q \tag{25}$$

and

$$\dot{E}_q' = \frac{E_{FD}}{T_{do}'} - \frac{E_q'}{T_{do}'} - \frac{(X_d - X_d')}{T_{do}'} i_d \tag{26}$$

The first model represents the rotor circuit and the field circuit by its time constant, while the second model represents only the field circuit by its time constant.

The quadrature and direct axis reactances changed if the Generating unit is connected to an infinite bus through a transmission line have a resistance R_e and a reactance X_e.

For Small Perturbation:

$$\Delta V_q = -r\Delta i_q - (X_d' + X_e)\,\Delta i_d + \Delta E_q' \tag{27}$$

$$\Delta V_d = -r\Delta i_d + (X_q' + X_e)\,\Delta i_q + \Delta E_d' \tag{28}$$

$$\Delta \dot{E}_q' = \frac{\Delta E_q'}{T_{do}'} - \frac{(X_d - X_d')}{T_{do}'}\Delta i_d + \frac{\Delta E_{FD}}{T_{do}'} \tag{29}$$

$$\Delta \dot{E}_d' = -\frac{\Delta E_d'}{T_{qo}'} + \frac{X_q - X_q'}{T_{qo}'}\,\Delta i_q \tag{30}$$

Figure 2 shows a block diagram describing Eqns. (29) and (30).

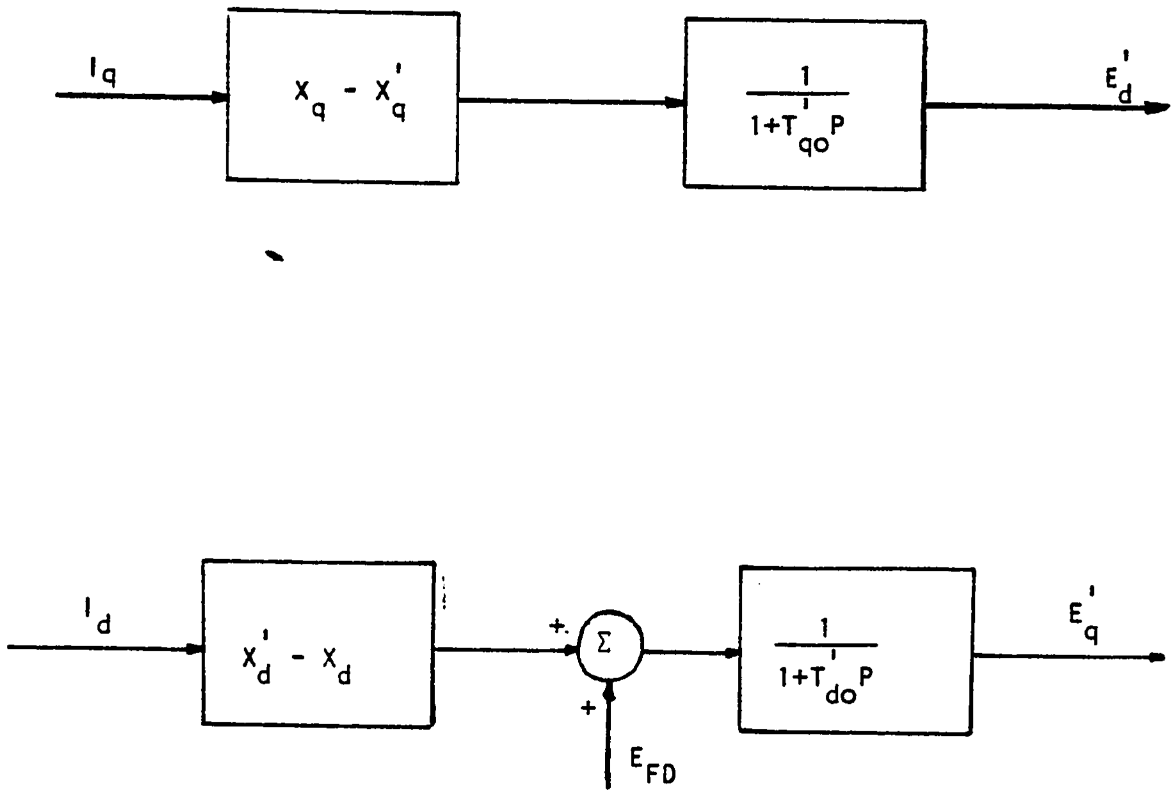


Figure 2. Block diagram for E_d' and E_q'.

Appendix A4-III

BEST FIT METHOD

Let a parameter X has a two values such as

$$X = \frac{K_{11}}{A_{11}} \qquad (1)$$

and

$$X = \frac{K_{12}}{A_{12}} \qquad (2)$$

where

X parameter

K_{11}, K_{12}, A_{11} and A_{12} are constants

From Eqns. (1) and (2) we can write

$$K_{11} = XA_{11} \qquad (3)$$

$$K_{12} = XA_{12} \qquad (4)$$

The Jacobain function is

$$J(X) = (K_{11} - XA_{11})^2 + (K_{12} - XA_{12})^2 \quad (5)$$

Equation (5) after differentiation

$$\therefore \frac{\partial J}{\partial X} = K_{11}A_1 - A_{11}^2 X + K_{12}A_{12} - A_{12}^2 X \quad (6)$$

To get the appropriate value for X equate Eqn. (6) by zero and solve for X.

$$\frac{\partial J}{\partial X} \leftarrow 0 \quad (7)$$

$$K_{11}A_{11} + K_{12}A_{12} - X\,(A_{11}^2 + A_{12}^2) = 0 \quad (8)$$

$$X = \frac{K_{11}A_1 + K_{12}A_{12}}{A_{12}^2 + A_{12}^2} \quad (9)$$

To generalize the idea, let X have three values such as

$$X = \frac{K_{11}}{A_{11}}$$

$$X = \frac{K_{12}}{A_{12}}$$

$$X = \frac{K_{13}}{A_{13}}$$

Using the same procedure we can come to appropriate value for X such as

$$X = \frac{K_{11}A_{11} + K_{12}A_{12} + K_{13}A_{13}}{A_{11}^2 + A_{12}^2 + A_{13}^2}$$

The general formula for certain parameter having n known value

$$X = \frac{\sum_{i=1}^{n} K_{1i} A_{1i}}{\sum_{i=1}^{n} A_{1i}^2}$$

Appendix A4-V

EXPRESSING ΔV_t IN TERMS OF MODAL VARIABLES "MACHINE REPRESENTED BY FIELD WINDING CIRCUIT IN THE DIRECT AXIS AND DAMPER WINDING CIRCUIT IN THE QUADRATURE AXIS"

It is desired to express ΔV_t in terms of the model variables. The synchronous machine terminal voltage is given by

$$V_t^2 = V_{t_d}^2 + V_{t_q}^2 \qquad (1)$$

This equation is linearized to obtain

$$\Delta V_t = \frac{V_{t_{do}}}{V_{to}} \Delta V_d + \frac{V_{t_{qo}}}{V_{to}} \Delta V_{t_q} \qquad (2)$$

ΔV_{t_d} and ΔV_{t_q} can be obtained from the internal machine equations

$$\Delta V_{t_q} = -X_d' \Delta i_d + \Delta E_q' \qquad (3)$$

$$\Delta V_{t_d} = X_q' \Delta i_q + \Delta E_d' \qquad (4)$$

Substituting (3) and (4) into (2)

$$\Delta V_t = \frac{V_{t_{do}}}{V_{to}} [X_q' \Delta i_q + \Delta E_d'] + \frac{V_{t_{qo}}}{V_{to}} [-X_d' \Delta i_d + \Delta E_q'] \qquad (5)$$

rearranging (5)

$$\Delta V_t = \frac{V_{t_{do}}}{V_{to}} X_q' \Delta i_q - \frac{V_{t_{qo}}}{V_{to}} X_d' \Delta i_d + \frac{V_{t_{do}}}{V_{to}} \Delta E_d' + \frac{V_{t_{qo}}}{V_{to}} \Delta E_q' \quad (6)$$

$\Delta E_d'$ and $\Delta E_q'$ can be obtained from the infinite bus equations

$$\Delta E_d' = -(X_q' + X_e)\,\Delta i_q + V_B \cos\delta_o\,\Delta\delta \quad (7)$$

$$\Delta E_q' = -(X_d' + X_e)\,\Delta i_d - V_B \sin\delta_o\,\Delta\delta \quad (8)$$

Substituting (7) and (8) into (6) and rearranging we get

$$\Delta V_t = [-\frac{V_{t_{do}}}{V_{to}} X_e]\,\Delta i_q + [\frac{V_{t_{qo}}}{V_{to}} X_e]\,\Delta i_d + [\frac{V_{t_{do}}}{V_{to}} V_B \cos\delta_o - \frac{V_{t_{qo}}}{V_{to}} V_B \sin\delta_o]\,\Delta\delta$$

(9)

$$\Delta V_t = k_{12}\Delta i_q + k_{13}\Delta i_d + k_{14}\Delta\delta$$

Where k_{12} is the change in the terminal voltage V_t for a small change in the quadrature axis current at constant rotor angle and direct axis current or

$$k_{12} = \frac{\Delta V_t}{\Delta i_q}\Big|_{\substack{\delta=\delta_o \\ I_d=I_{do}}}$$

$$k_{12} = -\frac{V_{t_{do}}}{V_{to}} X_e$$

k_{13} is the change in the terminal voltage V_t for a small change in the direct axis current at constant rotor angle and quadrature axis current or

$$k_{13} = \frac{\Delta V_t}{\Delta i_d}\Big|_{\substack{\delta=\delta_o \\ I_q=I_{qo}}}$$

$$k_{13} = \frac{V_{t_{qo}}}{V_{to}} X_e$$

and k_{14} is the change in the terminal voltage V_t for a small change in the rotor angle at constant direct axis current and quadrature axis current or

$$k_{14} = \frac{\Delta V_t}{\Delta \delta}\Big|_{\substack{I_q = I_{qo} \\ I_d=I_{do}}}$$

$$k_{14} = \frac{V_{t_{do}}}{V_{to}} V_B \cos\delta_o - \frac{V_{t_{qo}}}{V_{to}} V_B \sin\delta_o$$

If the machine is represented only by the field winding in the direct axis. In this case Eqn. (3) still the same and Eqn. (4) is replaced by

$$\Delta V_d = X_q \, \Delta i_q \tag{11}$$

Follow the same procedure as previous we obtain

$$\Delta V_t = \frac{V_{t_{do}}}{V_{to}} X_q \Delta i_q - \frac{V_{t_{qo}}}{V_{to}} X_d' \, \Delta i_d + \frac{V_{t_{qo}}}{V_{to}} \Delta E_q' \tag{12}$$

Equation (8) still the same and Eqn. (7) replaced by

$$0 = -(X_q + X_e) \, \Delta i_q + V_B \cos \delta_o \, \Delta\delta \tag{13}$$

Substituting Eqns. (8) and (13) into (12)

$$\Delta V_t = \frac{V_{t_{do}}}{V_{to}} X_q \left(\frac{V_B \cos \delta_o}{X_q + X_e} \right) \Delta\delta - \frac{V_{t_{qo}}}{V_{to}} [(X_d' + X_e) \, \Delta i_d - V_B \sin \delta_o \, \Delta\delta] - \frac{V_{t_{qo}}}{V_{to}} X_d' \Delta i_d \tag{14}$$

$$\Delta V_t = \frac{V_{t_{qo}}}{V_{to}} X_e \, \Delta i_d + \left[\frac{V_{t_{do}} \, X_q \, V_B \, \cos \delta_o}{V_{to}(X_q + X_e)} - \frac{V_{t_{qo}}}{V_{to}} V_B \sin \delta_o \right] \Delta\delta \tag{15}$$

rearranging Eqn. (15)

$$\Delta V_t = \frac{V_{t_{qo}}}{V_{to}} X_e \Delta i_d + \left[\frac{V_{t_{do}} V_{qo} X_q}{V_{to}(X_q + X_e)} - \frac{V_{t_{qo}} V_{do}}{V_{to}} \right] \Delta\delta \tag{16}$$

$$\Delta V_t = k_{12} \, \Delta i_d + k_{13} \Delta\delta \tag{17}$$

where k_{12} is the change in the terminal voltage V_t for a small change in the direct axis current at constant rotor angle or

$$k_{12} = \frac{\Delta V_t}{\Delta i_d}\Big|_{\delta=\delta_o}$$

$$k_{12} = \frac{V_{t_{qo}}}{V_{to}} X_e$$

and k_{13} is the change in the terminal voltage V_t for a small change in the rotor angle at constant direct axis current or

$$k_{13} = \frac{\Delta V_t}{\Delta \delta}\Big|_{I_d = I_{do}}$$

$$K_{13} = \frac{V_{t_{do}} V_{qo}}{V_{to}} \frac{X_q}{X_q + X_e} - \frac{V_{t_{qo}} V_{do}}{V_{to}}$$

PROGRAM LISTING

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C ***********************************************************************
C * GABER S. S. AHMED - MSEE CANDIDATE - FALL 1985                      *
C * A.H.EL-ABIAD - MAJOR PROFESSOR                                      *
C * MSEE - THESIS COMPUTER PROGRAM                                      *
C *                                                                     *
C * THIS PROGRAM BUILDS STATE MATRIX OF A COMPLETE GENERATING           *
C * UNIT OF ORDER 13*13 AND THEN REDUCES THE 13*13 TO 2*2,              *
C * 3*3 ,4*4 &5*5 USING MODAL REDUCTION METHOD,ALSO                     *
C * CALCULATES THE R.M.S. ERROR FOR EACH REDUCED ORDER MODEL            *
C * THE INPUT IS CONSIDERED TO BE THE CHNAGE IN THE                     *
C * LOAD POWER                                                          *
C *                                                                     *
C ***********************************************************************
      DIMENSION X1(13,13),R2(13,13),R1(13,13)
      REAL IQ(803),ID(803),DEL(803),W(803),EFD(803),TIME(803)
      REAL DEL2(803),W2(803),EFD2(803),ID2(803),W21(803),IQ2(803)
      REAL DEL3(803),IQ3(803),ID3(803),W3(803),DEL31(803),EFD3(803)
      REAL ID31(803),W31(803),IQ31(803)
      REAL ID4(803),EFD4(803),DEL4(803),W4(803),DEL41(803),W41(803)
      REAL ID41(803),IQ4(803)
      REAL IQ5(803),ID5(803),DEL5(803),W5(803),EFD5(803)
      REAL DDEL2(803),DW2(803),DFD2(803),DID2(803),DW21(803)
      REAL DDEL3(803),DIQ3(803),DID3(803),DW3(803),DDEL31(803)
      REAL DID31(803),DW31(803),DIQ31(803),DFD3(803),DIQ2(803)
      REAL DID4(803),DFD4(803),DDEL4(803),DW4(803),DDEL41(803)
      REAL DW41(803),DID41(803),DIQ4(803)
      REAL DIQ5(803),DID5(803),DDEL5(803),DW5(803),DFD5(803)
      COMPLEX  LAMDR3(3),LAMDR4(4),LAMDR5(5),TR3(3,3),TR4(4,4)
      COMPLEX  TR5(5,5),LAMDR2(2),TR2(2,2),CP1(5,5)
      COMPLEX  C(5,13),B(13,3),MR(5,5),GAMA(5,5),CP2(13,13)
      COMPLEX  T(5,13),A(13,13),A1(13,13)
      COMPLEX  ARR(5,5),BB(5,1),BB1(5,1),MRR(5,5),BR(5),AR(5,5)
      COMPLEX*8 LAMBDA(13),H(13,13),HL(13,13)
      INTEGER*4 INT(13)
      LOGICAL*1 INTH(13)
      READ(5,*) M
      READ(5,*) XQ,XD,XMQ,XMD,XKQ,XKD,XFD,XL
      READ(5,*) RA,RFD,RKD,RKQ,RL,CURQO,CURDO,CURFO
      READ(5,*) VB,VQO,VDO,VTO,SO,WB,CONSM,RG,TG,TT,DAMP
      READ(5,*) CONKE,CONKA,CONKR,CONKS,TE,TA,TR,TS
C SET UP THE WORKING MATRICES BY ZEROS.
      DO 1  J=1,M
      DO 1  K=1,M
 1    X1(J,K)=0.0
      DO 2  J=1,M
      DO 2  K=1,M
 2    R1(J,K)=0.0
      DO 3  J=1,M
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      DO 3  K=1,M
 3    R2(J,K)=0.0
C CHOOSE THE INTERESTED OUTPUT VARIABLES TO FORM
C THE OUTPUT MATRIX C.
      DO 4  J=1,5
      DO 4  K=1,M
 4    C(J,K)=(0.0,0.0)
      C(1,6)=(1.0,0.0)
      C(2,7)=(1.0,0.0)
      C(3,8)=(1.0,0.0)
      C(4,1)=(1.0,0.0)
      C(5,2)=(1.0,0.0)
C FORM THE MATRIX R1.
C MACHINE WINDINGS AND SHAFT REPRESENTATION.
      XD=XD+XL
      XQ=XQ+XL
      RA=RA+RL
      E1=(XQ-XD)*CURDO+XMD*CURFO
      E2=(XQ-XD)*CURQO
      E3=-XMQ*CURDO
      E4=XMD*CURQO
      E5=XMD*CURQO
      EPSIDO=-XD*CURDO+XMD*CURFO
      EPSIQO=-XQ*CURQO
      R1(1,1)=-RA
      R1(1,2)=-XD
      R1(1,4)=XMD
      R1(1,5)=R1(1,4)
      R1(1,6)=VB*SIN(SO)
      R1(1,7)=EPSIDO/WB
      R1(2,1)=XQ
      R1(2,2)=R1(1,1)
      R1(2,3)=-XMQ
      R1(2,6)=-VB*COS(SO)
      R1(2,7)=EPSIQO/WB
      R1(3,3)=RKQ
      R1(4,4)=RKD
      R1(5,5)=XMD
      R1(6,7)=-1.0
      R1(7,1)=E1/CONSM
      R1(7,2)=E2/CONSM
      R1(7,3)=E3/CONSM
      R1(7,4)=E4/CONSM
      R1(7,5)=E5/CONSM
      R1(7,7)=DAMP/CONSM
C EXCITER AND AUTOMATIC VOLTAGE REGULATOR REPRESENTATION.
      R1(5,8)=-1.0
      R1(7,12)=-1.0/CONSM
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      R1(8,8)=-1.0/TE
      R1(8,9)=CONKE/TE
      R1(9,9)=-1.0/TA
      R1(9,10)=-CONKA/TA
      R1(9,11)=-CONKA/TA
      R1(10,10)=-1.0/TR
      R1(11,8)=-CONKS/(TS*TE)
      R1(11,9)=(CONKS*CONKE)/(TS*TE)
      R1(11,11)=-1.0/TS
      R1(10,1)=(CONKR*VQO*RL-CONKR*VDO*XL)/(TR*VTO)
      R1(10,2)=(CONKR*VQO*XL+CONKR*VDO*RL)/(TR*VTO)
      R1(10,6)=(-CONKR*VQO*VB*SIN(SO)+CONKR*VDO*VB*COS(SO))/(TR*VTO)
      R1(10,7)=(CONKR*CURDO*XL*VQO-CONKR*XL*CURQO*VDO)/(WB*TR*VTO)
C TURBINE AND GOVERNOR REPRESENTATION.
      R1(12,12)=-1.0/TT
      R1(12,13)=1.0/TT
      R1(13,13)=-1.0/TG
      R1(13,7)=-1.0/(RG*TG)
C FORM THE MATRIX X1.
C MACHINE WINDINGS AND SHAFT REPRESENTATION.
      X1(1,1)=-XQ
      X1(1,3)=XMQ
      X1(2,2)=-XD
      X1(2,4)=XMD
      X1(2,5)=X1(2,4)
      X1(3,1)=-XMQ
      X1(3,3)=XKQ
      X1(4,2)=-XMD
      X1(4,4)=XKD
      X1(4,5)=XMD
      X1(5,2)=-XMD*XMD/RFD
      X1(5,4)=XMD*XMD/RFD
      X1(5,5)=XMD*XFD/RFD
      X1(6,6)=WB
      X1(7,7)=WB
C EXCITER AND AUTOMATIC VOLTAGE REGULATOR REPRESENTATION.
      X1(8,8)=-WB
      X1(9,9)=-WB
      X1(10,10)=-WB
      X1(11,11)=-WB
C TURBINE AND GOVERNOR REPRESENTATION.
      X1(12,12)=-WB
      X1(13,13)=-WB
      X1(10,1)=(XL*CONKR*VQO)/(TR*VTO)
      X1(10,2)=(XL*CONKR*VDO)/(TR*VTO)
      CALL INVERT(X1,M,0.001)
      CALL MATMU1(X1,R1,R2,M,M,M)
      DO 6 J=1,M
```

```
      DO 6 K=1,M
 6    R2(J,K)=-WB*R2(J,K)
C PUT THE MATRIX A IN A COMPLEX FORM AND CALL THE EIGENVALUES
C AND THE EIGENVECTORS SUBROUTINE.
C LAMBDA      EIGENVALUES OF THE MATRIX A.
C  A          REPLACED BY THE EIGENVECTORS MATRIX AFTER CALLING.
      DO 7 J=1,M
      DO 7 K=1,M
 7    A(J,K)=CMPLX(R2(J,K),0.0)
      PRINT 8
 8    FORMAT('1',2X,'A MATRIX OF 13TH ORDER SYSTEM',/)
      DO 9 J=1,M
 9    WRITE(6,108)(R2(J,K),K=1,M)
      CALL EIGCS(A,M,M,M,LAMBDA,39,NCAL,H,M,M,HL,M,M,INT,INTH)
C STORE THE EIGENVECTORS MATRIX.
      DO 12 J=1,M
      DO 12 K=1,M
 12   A1(J,K)=A(J,K)
C INVERT THE EIGENVECTORS MATRIX.
      CALL INVER(A1,M,0.001)
C  FORM THE MATRIX B(13,3).
      DO 17 J=1,M
      DO 17 K=1,3
 17   B(J,K)=(0.0,0.0)
      B1=1.0/CONSM
      B2=CONKA/TA
      B3=1.0/TG
      B(7,1)=CMPLX(B1,0.0)
      B(9,2)=CMPLX(B2,0.0)
      B(13,3)=CMPLX(B3,0.0)
C
C SUBROUTINE TFACTO CALCULATES THE INPUT-OUTPUT INDICES.
      CALL TFACTO(T,C,A,A1,B,LAMBDA,M,1)
C
C SUBROUTINE OUTPUT CALCULATES THE OUTPUTS
C OF THE COMPLETE SYSTEM.
      CALL OUTPUT(T,IQ,ID,DEL,W,EFD,LAMBDA,M)
C
C SUBROUTINE ABSTAR CALCULATES SECOND ORDER REDUCED MODELS
C USING MODES 6,7(IMPORTANT VARIABLES ARE DELTA  AND  W).
      CALL ABSTAR(2,6,7,0,0,0,6,7,0,0,0,A,A1,LAMBDA,B,M,LAMDR2,TR2,
     *INTH,H,HL,ARR,BB,BB1,MRR,BR,AR,CP1,MR,GAMA,2)
C
C CALCULATION OF THE OUTPUT OF THE SECOND REDUCED ORDER MODELS.
      CALL OUT2(TR2,DEL2,W2,LAMDR2,2)
C
C SUBROUTINE ABSTAR CALCULATES SECOND ORDER REDUCED MODELS
C USING MODES 6,7(IMPORTANT VARIABLES ARE   IQ  AND  W  ).
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```
      CALL ABSTAR(2,6,7,0,0,0,1,7,0,0,0,A,A1,LAMBDA,B,M,LAMDR2,TR2,
     *INTH,H,HL,ARR,BB,BB1,MRR,BR,AR,CP1,MR,GAMA,2)
C
C CALCULATION OF THE OUTPUT OF THE SECOND REDUCED ORDER MODELS.
      CALL OUT2(TR2,IQ2,W21,LAMDR2,2)
C
C SUBROUTINE ABSTAR CALCULATES SECOND ORDER REDUCED MODELS
C USING MODES 6,7(IMPORTANT VARIABLES ARE    EFD  AND  ID  ).
      CALL ABSTAR(2,6,7,0,0,0,2,8,0,0,0,A,A1,LAMBDA,B,M,LAMDR2,TR2,
     *INTH,H,HL,ARR,BB,BB1,MRR,BR,AR,CP1,MR,GAMA,2)
C
C CALCULATION OF THE OUTPUT OF THE SECOND REDUCED ORDER MODELS.
      CALL OUT2(TR2,ID2,EFD2,LAMDR2,2)
C
C SUBROUTINE ABSTAR CALCULATES THIRD ORDER REDUCED MODELS
C USING MODES 5,6,7(IMPORTANT VARIABLES ARE IQ & DELTA AND W).
      CALL ABSTAR(3,5,6,7,0,0,1,6,7,0,0,A,A1,LAMBDA,B,M,LAMDR3,TR3,
     *INTH,H,HL,ARR,BB,BB1,MRR,BR,AR,CP1,MR,GAMA,3)
C
C CALCULATION OF THE OUTPUT OF THE THIRD REDUCED ORDER MODELS.
      CALL OUT3(TR3,IQ3,DEL3,W3,LAMDR3,3)
C
C SUBROUTINE ABSTAR CALCULATES THIRD ORDER REDUCED MODELS
C USING MODES 5,6,7(IMPORTANT VARIABLES ARE  ID  & IQ  AND EFD).
      CALL ABSTAR(3,5,6,7,0,0,1,2,6,0,0,A,A1,LAMBDA,B,M,LAMDR3,TR3,
     *INTH,H,HL,ARR,BB,BB1,MRR,BR,AR,CP1,MR,GAMA,3)
C
C CALCULATION OF THE OUTPUT OF THE THIRD REDUCED ORDER MODELS.
      CALL OUT3(TR3,IQ31,ID3,DEL31,LAMDR3,3)
C
C SUBROUTINE ABSTAR CALCULATES THIRD ORDER REDUCED MODELS
C USING MODES 5,6,7(IMPORTANT VARIABLES ARE  ID  &  W  AND EFD).
      CALL ABSTAR(3,2,3,5,0,0,2,8,7,0,0,A,A1,LAMBDA,B,M,LAMDR3,TR3,
     *INTH,H,HL,ARR,BB,BB1,MRR,BR,AR,CP1,MR,GAMA,3)
C
C CALCULATION OF THE OUTPUT OF THE THIRD REDUCED ORDER MODELS.
      CALL OUT3(TR3,ID31,EFD3,W31,LAMDR3,3)
C
C SUBROUTINE ABSTAR CALCULATES FOURTH ORDER REDUCED MODELS
C USING MODES 2,3,6,7(IMPORTANT VARIABLES ARE DELTA & EFD & ID
C AND  W  ).
      CALL ABSTAR(4,3,2,6,7,0,2,8,6,7,0,A,A1,LAMBDA,B,M,LAMDR4,TR4,
     *INTH,H,HL,ARR,BB,BB1,MRR,BR,AR,CP1,MR,GAMA,4)
C
C CALCULATION OF THE OUTPUTS OF THE FOURTH REDUCED ORDER MODELS.
      CALL OUT4(TR4,ID4,EFD4,DEL4,W4,LAMDR4,4)
C
C SUBROUTINE ABSTAR CALCULATES FOURTH ORDER REDUCED MODELS
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```
C USING MODES 2,3,6,7(IMPORTANT VARIABLES ARE DELTA &IQ &ID
C AND W).
      CALL ABSTAR(4,2,3,6,7,0,1,2,6,7,0,A,A1,LAMBDA,B,M,LAMDR4,TR4,
     *INTH,H,HL,ARR,BB,BB1,MRR,BR,AR,CP1,MR,GAMA,4)
C
C CALCULATION OF THE OUTPUTS OF THE FOURTH REDUCED ORDER MODELS.
      CALL OUT4(TR4,IQ4,ID41,DEL41,W41,LAMDR4,4)
C
C SUBROUTINE ABSTAR CALCULATES FIFTH ORDER REDUCED MODEL
C USING MODES 2,3,5,6,7(IMPORTANT VARIABLES ARE IQ &ID &EFD
C & DELTA AND W).
      CALL ABSTAR(5,5,2,3,6,7,1,2,8,6,7,A,A1,LAMBDA,B,M,LAMDR5,TR5,
     *INTH,H,HL,ARR,BB,BB1,MRR,BR,AR,CP1,MR,GAMA,5)
C
C CALCULATION OF THE OUTPUTS OF THE FIFTH REDUCED ORDER MODELS.
      CALL OUT5(TR5,IQ5,ID5,EFD5,DEL5,W5,LAMDR5,5)
C
C SUBROUTINE ERROR CALCULATES THE ERROR OF THE REDUCED SYSTEM
      CALL ERROR(IQ,IQ2,IQ3,IQ31,IQ4,IQ5,ID,ID2,ID3,ID31,
     *ID4,ID41,ID5,DEL,DEL2,DEL3,DEL31,DEL4,DEL41,DEL5,
     *W,W2,W21,W3,W31,W4,W41,W5,EFD,EFD2,EFD3,EFD4,EFD5,
     *DIQ2,DIQ31,DIQ3,DIQ4,DIQ5,DID2,DID3,DID31,DID4,DID41,DID5
     *,DDEL2,DDEL3,DDEL31,DDEL4,DDEL41,DDEL5,DW,DW2,
     *DW21,DW3,DW31,DW4,DW41,DW5,DFD2,DFD3,DFD4,DFD5)
      PRINT 32
 32   FORMAT(5X,/,' THE ROTOR ANGLE OUTPUT  ')
      WRITE(6,34)
      DO 33 I=1,801
      TIME(I)=(I-1)*.005
 33   WRITE(10,35)TIME(I),DEL(I),DEL2(I),DEL3(I),DEL4(I),DEL5(I)
 35   FORMAT(2X,F9.4,1x,F9.4,1X,F9.4,1X,F9.4,1X,F9.4,1x,F9.4)
 34   FORMAT('1',T4,'TIME',T12,'DELTA',T20,'DEL-SEC',T28,'DEL-THI'
     *,T36,'DEL-FOU',T45,'DEL-FIF')
      PRINT 36
 36   FORMAT(5X,/,' THE ROTOR ANGLE ERROR  ')
      WRITE(6,37)
      DO 38 I=1,801
      TIME(I)=(I-1)*.005
 38   WRITE(10,39)TIME(I),DDEL(I),DDEL2(I),DDEL3(I),DDEL4(I),DDEL5(I)
 39   FORMAT(2X,F9.4,1x,F9.4,1X,F9.4,1X,F9.4,1X,F9.4,1x,F9.4)
 37   FORMAT('1',T4,'TIME',T12,'DELTA',T20,'DEL-SEC',T28,'DEL-THI'
     *,T36,'DEL-FOU',T45,'DEL-FIF')
 108  FORMAT(1X,/,13F10.4)
 110  FORMAT(1X,/,3(F10.4,F10.4))
 120  FORMAT(1X,7(F8.4,F8.4))
      STOP
      END
C SUBROUTINE TO MULTIPLY TWO REAL MATRICES.
```

```
      SUBROUTINE MATMU1(AB,AKT,CP,N,K,M)
      DIMENSION AB(N,K),AKT(K,M),CP(N,M)
      DO 50 I=1,N
      DO 50 J=1,M
      CP(I,J)=0.0
      DO 50 L=1,K
   50 CP(I,J)=CP(I,J)+AB(I,L)*AKT(L,J)
      RETURN
      END
C SUBROUTINE TO MULTIPLY TWO COMPLEX MATRICES.
      SUBROUTINE MATMU2(AB,AKT,CP,N,K,M)
      COMPLEX AB(N,K),AKT(K,M),CP(N,M)
      DO 50 I=1,N
      DO 50 J=1,M
      CP(I,J)=0.0
      DO 50 L=1,K
   50 CP(I,J)=CP(I,J)+AB(I,L)*AKT(L,J)
      RETURN
      END
C SUBROUTINE TO INVERT A REAL MATRIX A(N,N) BY GYRATING IT.
      SUBROUTINE INVERT(A,N,F)
      DIMENSION A(N,N),S(13,13)
      DO 81 I=1,N
   81 S(I,I)=A(I,I)
      DO 82 I=1,N
      CALL GMAX(S,N,IK,XLARGE)
      S(IK,IK)=0.0
      IF(ABS(XLARGE).LT.F)GO TO 83
      CALL GYRATE(A,N,IK)
      DO 82 I1=1,N
      IF(ABS(S(IK,IK)).EQ.0.0)GO TO 82
      S(I1,I1)=A(I1,I1)
   82 CONTINUE
   83 RETURN
      END
      SUBROUTINE GMAX(X,N,J,XLARGE)
      DIMENSION X(N,N)
      XLARGE=ABS(X(1,1))
      J=1
      IF(N.EQ.1)GO TO 84
      DO 85 I=2,N
      IF(ABS(X(I,I)).LE.XLARGE)GO TO 85
      XLARGE=ABS(X(I,I))
      J=I
   85 CONTINUE
   84 RETURN
      END
      SUBROUTINE GYRATE(A,N,I)
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```
      DIMENSION A(N,N)
      A(I,I)=1.0/A(I,I)
      DO 86 J=1,N
      IF(J.EQ.I)GO TO 86
      A(I,J)=-A(I,J)*A(I,I)
   86 CONTINUE
      DO 87 K=1,N
      IF(K.EQ.I)GO TO 87
      DO 88 J=1,N
      IF(J.EQ.I)GO TO 88
      A(K,J)=A(K,J)+A(K,I)*A(I,J)
   88 CONTINUE
   87 CONTINUE
      DO 89 K=1,N
      IF(K.EQ.I)GO TO 89
      A(K,I)=A(K,I)*A(I,I)
   89 CONTINUE
      RETURN
      END
C     THIS SUBROUTINE CALCULATES INVERSE OF A MATRIX.
C     A  IS THE INPUT MATRIX.
C     N IS THE ORDER OF THE MATRIX.
C     E  IS A SMALL VALUE IN THE ORDER 10(-3).
C     USAGE CALL INVER(A,N,E).
      SUBROUTINE INVER(A,N,E)
      COMPLEX A
      DIMENSION A(N,N),B(15)
      DO 5 I=1,N
      B(I)=0.0
    5 CONTINUE
      M=0
   11 BIG=0.0
      I=0
   20 I=I+1
      IF(I.GT.N)GO TO 33
      IF(B(I).EQ.I)GO TO 20
      IF((CABS(A(I,I))).GE.BIG)GO TO 22
      GO TO 20
   22 L=I
      BIG=(CABS(A(L,L)))
      K=L
      GO TO 20
   33 IF((CABS(A(K,K))).LE.E)GO TO 34
      CALL GYRAT(A,N,K)
      B(K)=K
      M=M+1
      IF(M.EQ.N)GO TO 35
      GO TO 11
```

```
   34  PRINT 39
   39  FORMAT(' ',15X,'A MAY BE SINGULAR AND OF RANK M')
   40  WRITE(6,41) M
   41  FORMAT(' ',10X,'M=',I2)
   35  RETURN
        END
C THIS SUBROUTINE IS USED ONLY TO GYRATE
C ROW AND COLUMN OF A MATRIX.
C SUBROUTINE GYRATE  (A,N,I).
C A IS THE INPUT MATRIX.
C N IS THE ORDER OF THE MATRIX A.
C K IS THE AXIS ON WHICH GYRATION IS TO BE DONE.
C USAGE CALL GYRATE(A,N,K).
        SUBROUTINE GYRAT(A,N,K)
        COMPLEX A
        DIMENSION A(N,N)
        A(K,K)=1.0/A(K,K)
        DO 1 L=1,N
        IF(L.EQ.K)GO TO 1
        A(K,L)=-A(K,L)*A(K,K)
    1  CONTINUE
        DO 2 I=1,N
        IF(I.EQ.K)GO TO 2
        DO 9 J=1,N
        IF(J.EQ.K)GO TO 9
        A(I,J)=(A(I,J)+A(I,K)*A(K,J))
    9  CONTINUE
    2  CONTINUE
        DO 3 II=1,N
        IF(II.EQ.K)GO TO 3
        A(II,K)=A(II,K)*A(K,K)
    3  CONTINUE
        RETURN
        END
C
C SUBROUTINE TO GET THE EIGENVALUES AND THE EIGENVECTORS.
C
        SUBROUTINE EIGCS(A,IA,JA,M,W,IW,NCAL,H,IH,JH,HL,IHL,
       *JHL,INT,INTH)
        COMPLEX*8 A(IA,JA),H(IH,JH),HL(IHL,JHL),W(IW),
       &MULTI,SHIFTI,CONCOS,GPV,GPS,CSQRT,TEMP2,SIN,
       &COS,TEMP,TEMP1,CMPLX,CONJG,CONSIN,SHIFT(3)
        INTEGER*4 INT(M),R,RP1,RP2
        LOGICAL*1 INTH(M),TWICE
        N=M
        I2=M+M
        NCAL=N
        IF (N.NE.1) GO TO 10
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      W(1)=A(1,1)
      A(1,1)=(1.,0.)
      GO TO 580
10    ICOUNT=0
      SHIFT(1)=(0.,0.)
      IF (N.NE.2) GO TO 40
20    TEMP=(A(1,1)+A(2,2)+CSQRT((A(1,1)+A(2,2))**2-(4.,0.)*
     &(A(2,2)*A(1,1)-A(2,1)*A(1,2))))/(2.,0.)
      IF (REAL(TEMP).NE.0..OR.AIMAG(TEMP).NE.0.) GO TO 30
      W(M)=SHIFT(1)
      W(M-1)=A(1,1)+A(2,2)+SHIFT(1)
      GO TO 380
30    W(M)=TEMP+SHIFT(1)
      W(M-1)=(A(2,2)*A(1,1)-A(2,1)*A(1,2))/(W(M)-SHIFT(1))+SHIFT(1)
      GO TO 380
C  REDUCE MATRIX A TO HESSENBERG FORM.
40    NM2=N-2
      DO 150 R=1,NM2
      RP1=R+1
      RP2=R+2
      ABIG=0.
      INTR=RP1
      DO 50 I=RP1,N
      ABSSQ=CABS(A(I,R))**2
      IF (ABSSQ.LE.ABIG) GO TO 50
      INTR=I
      ABIG=ABSSQ
50    CONTINUE
      INTER=INTR
      INT(R)=INTR
      IF (INTER.EQ.RP1) GO TO 80
      IF (ABIG.EQ.0.) GO TO 150
      DO 60 I=R,N
      TEMP=A(RP1,I)
      A(RP1,I)=A(INTER,I)
60    A(INTER,I)=TEMP
      DO 70 I=1,N
      TEMP=A(I,RP1)
      A(I,RP1)=A(I,INTER)
70    A(I,INTER)=TEMP
80    GPV=A(RP1,R)
      DO 90 I=RP2,N
      W(M+I)=A(I,R)/GPV
90    A(I,R)=W(M+I)
      DO 110 I=1,RP1
      TEMP=(0.,0.)
      DO 100 J=RP2,N
100   TEMP=TEMP+A(I,J)*W(M+J)
```

```
110   A(I,RP1)=A(I,RP1)+TEMP
      GPV=A(RP1,RP1)
      DO 130 I=RP2,N
      TEMP=(0.,0.)
      DO 120 J=RP2,N
120   TEMP=TEMP+A(I,J)*W(M+J)
130   A(I,RP1)=A(I,RP1)+TEMP-W(M+I)*GPV
      DO 140 J=RP2,N
      GPV=A(RP1,J)
      DO 140 I=RP2,N
140   A(I,J)=A(I,J)-W(M+I)*GPV
150   CONTINUE
C CALCULATE EPSILON.
      EPS=0.
      DO 160 I=1,N
160   EPS=EPS+CABS(A(1,I))
      DO 180 I=2,N
      SUM=0.
      IM1=I-1
      DO 170 J=IM1,N
170   SUM=SUM+CABS(A(I,J))
180   IF (SUM.GT.EPS) EPS=SUM
      EPS=SQRT(FLOAT(N))*EPS*1.E-10
      IF (EPS.LT.1.E-10) EPS=1.E-10
      DO 190 I=1,N
      DO 190 J=1,N
190   H(J,I)=A(J,I)
200   IF (N.NE.1) GO TO 210
      W(M)=A(1,1)+SHIFT(1)
      GO TO 380
210   IF (N.EQ.2) GO TO 20
220   MN1=M-N+1
      IF (REAL(A(N,N)).NE.0..OR.AIMAG(A(N,N)).NE.0.)
     &IF  (ABS(REAL(A(N,N-1)/A(N,N)))+ABS(AIMAG(A(N,N-1)
     &/A(N,N)))-1.E-7) 240,240,230
230   IF(ABS(REAL(A(N,N-1)))+ABS(AIMAG(A(N,N-1))).GE.EPS) GO TO 250
240   W(MN1)=A(N,N)+SHIFT(1)
      ICOUNT=0
      N=N-1
      GO TO 210
C DETERMINE SHIFT.
250   SHIFT(2)=(A(N-1,N-1)+A(N,N)+CSQRT((A(N-1,N-1)+A(N,N))
     &**2-(4.,0.)*(A(N,N)*A(N-1,N-1)-A(N,N-1)*A(N-1,N))))/(2.,0.)
      IF (REAL(SHIFT(2)).NE.0..OR.AIMAG(SHIFT(2)).NE.0.) GO TO 260
      SHIFT(3)=A(N-1,N-1)+A(N,N)
      GO TO 270
260   SHIFT(3)=(A(N,N)*A(N-1,N-1)-A(N,N-1)*A(N-1,N))/SHIFT(2)
270   IF (CABS(SHIFT(2)-A(N,N)).LT.CABS(SHIFT(3)-A(N,N))) GO TO 280
```

```
      INDEX=3
      GO TO 290
280   INDEX=2
290   IF (CABS(A(N-1,N-2)).GE.EPS) GO TO 300
      W(MN1)=SHIFT(2)+SHIFT(1)
      W(MN1+1)=SHIFT(3)+SHIFT(1)
      ICOUNT=0
      N=N-2
      GO TO 200
300   SHIFT(1)=SHIFT(1)+SHIFT(INDEX)
      SHIFTI=SHIFT(INDEX)
      DO 310 I=1,N
310   A(I,I)=A(I,I)-SHIFTI
C     PERFORM GIVENS ROTATIONS, QR ITERATES.
      IF (ICOUNT.LE.10) GO TO 320
      NCAL=M-N
      GO TO 380
320   NM1=N-1
      TEMP1=A(1,1)
      TEMP2=A(2,1)
      DO 370 R=1,NM1
      RP1=R+1
      RT1=REAL(TEMP1)
      IF (ABS(RT1).LT.1.E-35) RT1=0.
      RT2=REAL(TEMP2)
      IF (ABS(RT2).LT.1.E-35) RT2=0.
      AT1=AIMAG(TEMP1)
      IF (ABS(AT1).LT.1.E-35) AT1=0.
      AT2=AIMAG(TEMP2)
      IF (ABS(AT2).LT.1.E-35) AT2=0.
      TEMP1=CMPLX(RT1,AT1)
      TEMP2=CMPLX(RT2,AT2)
      RHO=SQRT(RT1**2+AT1**2+RT2**2+AT2**2)
      IF (RHO.EQ.0.) GO TO 370
      COS=TEMP1/RHO
      SIN=TEMP2/RHO
      CONCOS=CONJG(COS)
      CONSIN=CONJG(SIN)
      INDEX=MAX0(R-1,1)
      DO 330 I=INDEX,N
      GPV=A(RP1,I)
      GPS=A(R,I)
      TEMP=CONCOS*GPS+CONSIN*GPV
      A(RP1,I)=-SIN*GPS+COS*GPV
330   A(R,I)=TEMP
      IF (R.EQ.NM1) GO TO 340
      TEMP1=A(RP1,RP1)
      TEMP2=A(R+2,RP1)
```

```
340   DO 350 I=1,R
       GPV=A(I,RP1)
       GPS=A(I,R)
       TEMP=COS*GPS+SIN*GPV
       A(I,RP1)=-CONSIN*GPS+CONCOS*GPV
350   A(I,R)=TEMP
       INDEX=MIN0(R+2,N)
       DO 360 I=RP1,INDEX
       GPV=A(I,RP1)
       A(I,R)=SIN*GPV
360   A(I,RP1)=CONCOS*GPV
370   CONTINUE
       ICOUNT=ICOUNT+1
       GO TO 220
C     CALCULATE VECTORS
380   IF (NCAL.EQ.0) GO TO 580
       N=M
       NM1=N-1
       IF (N.NE.2) GO TO 390
       EPS=AMAX1(CABS(W(1)),CABS(W(2)))*1.E-6
       IF (EPS.LT.1.E-10) EPS=1.E-10
       H(1,1)=A(1,1)
       H(1,2)=A(1,2)
       H(2,1)=A(2,1)
       H(2,2)=A(2,2)
390   DO 570 L=1,NCAL
       DO 410 I=1,N
       DO 400 J=1,N
400   HL(J,I)=H(J,I)
410   HL(I,I)=HL(I,I)-W(L)
       DO 450 I=1,NM1
       W(M+I)=(0.,0.)
       INTH(I)=.FALSE.
       IP1=I+1
       IF (CABS(HL(IP1,I)).LE.CABS(HL(I,I))) GO TO 430
       INTH(I)=.TRUE.
       DO 420 J=I,N
       TEMP=HL(IP1,J)
       HL(IP1,J)=HL(I,J)
420   HL(I,J)=TEMP
430   IF (REAL(HL(I,I)).EQ.0..AND.AIMAG(HL(I,I)).EQ.0.) GO TO 450
       W(M+I)=-HL(IP1,I)/HL(I,I)
       MULTI=W(M+I)
       DO 440 J=IP1,N
440   HL(IP1,J)=HL(IP1,J)+MULTI*HL(I,J)
450   CONTINUE
       DO 460 I=1,N
460   W(I2+I)=(1.,0.)
```

```
      TWICE=.FALSE.
470   IF (REAL(HL(N,N)).EQ.0..AND.AIMAG(HL(N,N)).
     &.EQ.0.) HL(N,N)=EPS
      W(I2+N)=W(I2+N)/HL(N,N)
      DO 490 I=1,NM1
      K=N-I
      GPV=W(I2+K)
      DO 480 J=K,NM1
480   GPV=GPV-HL(K,J+1)*W(I2+J+1)
      IF (REAL(HL(K,K)).EQ.0..AND.AIMAG(HL(K,K))
     &.EQ.0.) HL(K,K)=EPS
490   W(I2+K)=GPV/HL(K,K)
      BIG=0.
      DO 500 I=1,N
      GPV=W(I2+I)
      SUM=ABS(REAL(GPV))+ABS(AIMAG(GPV))
500   IF (SUM.GT.BIG) BIG=SUM
      DO 510 I=1,N
510   W(I2+I)=W(I2+I)/BIG
      IF (TWICE) GO TO 530
      DO 520 I=1,NM1
      IP1=I+1
      IF (.NOT.INTH(I)) GO TO 520
      TEMP=W(I2+I)
      W(I2+I)=W(I2+IP1)
      W(I2+IP1)=TEMP
520   W(I2+IP1)=W(I2+IP1)+W(M+I)*W(I2+I)
      TWICE=.TRUE.
      GO TO 470
530   IF (N.EQ.2) GO TO 560
      NM2=N-2
      DO 550 I=1,NM2
      N1I=N-1-I
      NI1=N-I+1
      NNNN=N1I+1
      DO 540 J=NI1,N
540   W(I2+J)=H(J,N1I)*W(I2+NNNN)+W(I2+J)
      INDEX=INT(N1I)
      TEMP=W(I2+NNNN)
      W(I2+NNNN)=W(I2+INDEX)
550   W(I2+INDEX)=TEMP
560   DO 570 I=1,N
570   A(I,L)=W(I2+I)
580   RETURN
      END
C SUBROUTINE TO CALCULATE THE INPUT-OUTPUT PERFORMANCE INDICES.
      SUBROUTINE TFACTO(T,C,A,A1,B,LAMBDA,M,MM)
      COMPLEX A(M,M),LAMBDA(M)
```

```
      COMPLEX T(5,M),A1(M,M),C(5,M),B(M,3)
      COMPLEX SUM,SUM1,PROD,PROD1
      DO 9 K=1,5
      DO 10 J=1,M
      SUM=(0.0,0.0)
      DO 11 I=1,M
      PROD=C(K,I)*A(I,J)
 11   SUM=PROD+SUM
      SUM1=(0.0,0.0)
      DO 20 I=1,M
      PROD1=A1(J,I)*B(I,MM)
 20   SUM1=PROD1+SUM1
 10   T(K,J)=-SUM*SUM1/LAMBDA(J)
 9    CONTINUE
      PRINT 24,MM
 24   FORMAT('1',2X,'THE INPUT-OUTPUT PERFORMANCE INDEX TO ',I2,
     */)
      WRITE(6,106)
      DO 25 I=1,M
 25   WRITE(6,112)I,LAMBDA(I),T(1,I),T(2,I),T(3,I),T(4,I),T(5,I)
 112  FORMAT(2X,I2,F9.3,'J',F9.3,5X,F8.3,'J',F7.3,5X,F8.3,'J',
     *F7.3,5X,F8.3,'J',F7.3,5X,F8.3,'J',F7.3,5X,F8.3,'J',F7.3/)
 106  FORMAT(T3,'MODE',T12,'EIGENVALUE',T27,'RANK',T36,'DELTA',
     *T47,'RANK',T55,'FREQ',T69,'RANK',T77,'FIELD V',T90,'RANK',
     *T100,'IQ',T112,'RANK',T119,'ID'/)
      RETURN
      END
C SUBROUTINE TO CALCULATE THE OUTPUT OF THE COMPLETE SYSTEM.
      SUBROUTINE OUTPUT(T,IQ,ID,DEL,W,EFD,LAMDA,MM)
      COMPLEX T(5,MM),LAMDA(MM),B2
      REAL SUM1,SUM2,SUM3,SUM4,SUM5
      REAL IQ(803),ID(803),DEL(803),W(803),EFD(803),TIME(803)
      U=1
      DO 1 I=1,801
      TIME(I)=(I-1)*0.005
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      SUM4=0.0
      SUM5=0.0
      DO 2 J=1,MM
      B2=LAMDA(J)*TIME(I)
      IF(CABS(B2).GT.75)GO TO 5
      SUM1=SUM1+T(1,J)*(CMPLX(1.,0.)-CEXP(B2))*U
      SUM2=SUM2+T(2,J)*(CMPLX(1.,0.)-CEXP(B2))*U
      SUM3=SUM3+T(3,J)*(CMPLX(1.,0.)-CEXP(B2))*U
      SUM4=SUM4+T(4,J)*(CMPLX(1.,0.)-CEXP(B2))*U
      SUM5=SUM5+T(5,J)*(CMPLX(1.,0.)-CEXP(B2))*U
```

```
      GO TO 2
5     SUM1=SUM1+T(1,J)*(CMPLX(1.,0.))*U
      SUM2=SUM2+T(2,J)*(CMPLX(1.,0.))*U
      SUM3=SUM3+T(3,J)*(CMPLX(1.,0.))*U
      SUM4=SUM4+T(4,J)*(CMPLX(1.,0.))*U
      SUM5=SUM5+T(5,J)*(CMPLX(1.,0.))*U
2     CONTINUE
      DEL(I)=SUM1
      W(I)=SUM2
      EFD(I)=SUM3
      IQ(I)=SUM4
      ID(I)=SUM5
1     CONTINUE
      RETURN
      END
C SUBROUTINE TO CALCULATE THE OUTPUT OF SECOND ORDER REDUCED

      SUBROUTINE OUT2(TR,DEL1,W1,LAMDR,NM)
      COMPLEX TR(2,NM),LAMDR(NM)
      REAL DEL1(803),W1(803),TIME(803),SUM1,SUM2
      U=1
      DO 1 I=1,801
      TIME(I)=(I-1)*0.005
      SUM1=0.0
      SUM2=0.0
      DO 2 J=1,NM
      SUM1=SUM1+TR(1,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
2     SUM2=SUM2+TR(2,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
      DEL1(I)=SUM1
1     W1(I)=SUM2
      RETURN
      END
C SUBROUTINE TO CALCULATE THE OUTPUT OF 3RD ORDER REDUCED MODEL.
      SUBROUTINE OUT3(TR,EFD3,DEL1,W1,LAMDR,NM)
      COMPLEX TR(3,NM),LAMDR(NM)
      REAL SUM1,SUM2,SUM3
      REAL DEL1(803),W1(803),EFD3(803),TIME(803)
      U=1
      DO 1 I=1,801
      TIME(I)=(I-1)*0.005
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      DO 2 J=1,NM
      SUM1=SUM1+TR(1,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
      SUM2=SUM2+TR(2,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
2     SUM3=SUM3+TR(3,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
      EFD3(I)=SUM1
      DEL1(I)=SUM2
```

```
1     W1(I)=SUM3
      RETURN
      END
C SUBROUTINE TO CALCULATE THE OUTPUT OF 4TH ORDER REDUCED MODEL.
      SUBROUTINE OUT4(TR,ID1,EFD1,DEL1,W1,LAMDR,NM)
      COMPLEX TR(4,NM),LAMDR(NM)
      REAL SUM1,SUM2,SUM3,SUM4
      REAL EFD1(803),ID1(803),DEL1(803),W1(803),TIME(803)
      U=1
      DO 1 I=1,801
      TIME(I)=(I-1)*0.005
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      SUM4=0.0
      DO 2 J=1,NM
      SUM1=SUM1+TR(1,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
      SUM2=SUM2+TR(2,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
      SUM3=SUM3+TR(3,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
2     SUM4=SUM4+TR(4,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
      ID1(I)=SUM1
      EFD1(I)=SUM2
      DEL1(I)=SUM3
1     W1(I)=SUM4
      RETURN
      END
C SUBROUTINE TO CALCULATE THE OUTPUT OF 5TH ORDER REDUCED MODEL.
      SUBROUTINE OUT5(TR,IQ1,ID1,EFD1,DEL1,W1,LAMDR,NM)
      COMPLEX TR(5,NM),LAMDR(NM)
      REAL SUM1,SUM2,SUM3,SUM4,SUM5
      REAL IQ1(803),ID1(803),DEL1(803),W1(803),EFD1(803),TIME(803)
      U=1
      DO 1 I=1,801
      TIME(I)=(I-1)*0.005
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      SUM4=0.0
      SUM5=0.0
      DO 2 J=1,NM
      SUM1=SUM1+TR(1,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
      SUM2=SUM2+TR(2,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
      SUM3=SUM3+TR(3,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
      SUM4=SUM4+TR(4,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
2     SUM5=SUM5+TR(5,J)*(CMPLX(1.,0.)-CEXP(LAMDR(J)*TIME(I)))*U
      IQ1(I)=SUM1
      ID1(I)=SUM2
      EFD1(I)=SUM3
```

```
      DEL1(I)=SUM4
 1    W1(I)=SUM5
      RETURN
      END
c SUBROUTINE TO REDUCE THE COMPLETE SYSTEM.
      SUBROUTINE ABSTAR(NM,M1,M2,M3,M4,M5,N1,N2,N3,N4,N5,
     *A,A1,LAMBDA,B,M,LAMDR,TR,INTH,H,HL,ARR,BB,
     *BB1,MRR,BR,AR,CP1,MR,GAMA,NO)
      INTEGER AM(5),AN(5)
      COMPLEX A(M,M),A1(M,M),B(M,3),LAMBDA(M),MR(NM,NM),GAMA(NM,NM)
      COMPLEX MRR(NM,NM),BR(NM),AR(NM,NM),CP1(NM,NM)
      COMPLEX ARR(NM,NM),BB(NM,1),TR(NM,NM),BB1(NM,1)
      COMPLEX*8 H(NM,NM),HL(NM,NM),LAMDR(NM)
      LOGICAL*1 INTH(NM)
      INTEGER*4 INT(5)
      AM(1)=M1
      AM(2)=M2
      AM(3)=M3
      AM(4)=M4
      AM(5)=M5
      AN(1)=N1
      AN(2)=N2
      AN(3)=N3
      AN(4)=N4
      AN(5)=N5
      PRINT 111
 111  FORMAT('1',/,120('*'))
      PRINT 108
 108  FORMAT(/,2X,'THE RESULT OF THIS REDUCED MODEL ')
      PRINT 112
 112  FORMAT(/,2X,33('*'))
      WRITE(6,107)
      WRITE(6,203)(AM(I),I=1,NM)
      WRITE(6,109)
      WRITE(6,203)(AN(I),I=1,NM)
 203  FORMAT(5X,I6)
 107  FORMAT(/,2X,'MODES DOMINATED ARE')
 109  FORMAT(/,2X,'VARIABLES DOMINATED ARE')
C     SELECTion of THE REDUCED EIGENVECTOR MATRIX FROM THE MAIN
C     EIGENVECTOR MATRIX
C     THE COLUMNS IS CORRESPONDING TO EIGENVALUES
C     THE ROWS IS CORRESPONDING TO DOMINAT VARIABLES
      DO 10 I=1,NM
      II=AN(I)
      DO 10 J=1,NM
      MR(I,J)=(0.,0.)
      JJ=AM(J)
 10   MR(I,J)=A(II,JJ)
```

```
C STORE THE REDUCED EIGENVECTOR MATRIX
      DO 13 I=1,NM
      DO 13 J=1,NM
 13   MRR(I,J)=MR(I,J)
C INVERT THE REDUCED EIGENVECTOR MATRIX
      CALL INVER(MR,NM,0.0001)
C CONSTRUCT THE REDUCED EIGENVALUES MATRIX
      DO 16 I=1,NM
      DO 16 J=1,NM
 16   GAMA(I,J)=(0.0,0.0)
      DO 17 J=1,NM
      JJ=AM(J)
 17   GAMA(J,J)=LAMBDA(JJ)
      CALL MATMU2(GAMA,MR,CP1,NM,NM,NM)
      CALL MATMU2(MRR,CP1,AR,NM,NM,NM)
      PRINT 20
 20   FORMAT(/,2X,'THE MATRIX A FROM THE DETAILED MODEL ',/)
      DO 21 I=1,NM
 21   WRITE(6,116)(AR(I,J),J=1,NM)
C CALCULATION OF THE EIGENVALUES & EIGENVECTORS OF THE
C REDUCED system.
      CALL EIGCS(AR,NM,NM,NM,LAMDR,3*NM,NCAL,H,NM,NM,HL,
     *NM,NM,INT,INTH)
C STORE THE EIGENVECTOR MATRIX OF THE REDUCED A MATRIX
      PRINT 24
 24   FORMAT(5X,/,'    THE EIGENVALUES OF THE REDUCED A MATRIX')
      WRITE(6,123)(LAMDR(I),I=1,NM)
      DO 40 I=1,NM
 40   LAMDR(I)=(0.,0.)
      DO 41 J=1,NM
 41   LAMDR(J)=GAMA(J,J)
      PRINT 24
 24   FORMAT(5X,/,'    THE EIGENVALUES OF THE REDUCED A MATRIX')
      WRITE(6,123)(LAMDR(I),I=1,NM)
C CONSTRUCTION OF THE REDUCED MATRIX B.
      DO 33 I=1,NM
      II=AM(I)
      BB1(I,1)=(0.,0.)
      DO 33 J=1,M
 33   BB1(I,1)=BB1(I,1)+A1(II,J)*B(J,1)
      DO 34 I=1,NM
      BR(I)=(0.,0.)
      DO 34 J=1,NM
 34   BR(I)=BR(I)+MRR(I,J)*BB1(J,1)
      PRINT 27
 27   FORMAT(5X,/,'    THE REDUCED       B     MATRIX ')
      WRITE(6,123)(BR(I),I=1,NM)
C CONSTRUCTION OF THE T FACTOR FOR THE REDUCED SYSTEM
```

```
      DO 30 I=1,NM
      BB(I,1)=(0.,0.)
      DO 30 J=1,NM
 30   BB(I,1)=BB(I,1)+MR(I,J)*BR(J)
      DO 32 I=1,NO
      DO 32 J=1,NM
 32   TR(I,J)=-MRR(I,J)*BB(J,1)/LAMDR(J)
      PRINT 110
 110  FORMAT(//,2X,'THE RESULTs OF THIS REDUCED MODEL ARE FINISHED')
C WRITE THE REDUCED INPUT-OUTPUT PERFORMANCE INDICES.
      WRITE(6,106)
      DO 59 I=1,NM
 59   WRITE(6,155)I,LAMDR(I),(TR(J,I),J=1,NO)
 155  FORMAT('0',/I2,3X,F8.3,'J',F8.3,5(5X,F8.5,'J',F8.5))
 106  FORMAT('0','MODE',5X,'EIGENVALUE',10X,'     T-FACTORS')
 123  FORMAT(1X,F12.6,'+J',F12.6)
 116  FORMAT(2X,5(F10.5,F10.5))
      RETURN
      END
C SUBROUTINE TO CALCULATE THE ERROR OF THE REDUCED SYSTEM.
      SUBROUTINE ERROR(IQ,IQ2,IQ3,IQ31,IQ4,IQ5,ID,ID2,ID3,ID31,
     *ID4,ID41,ID5,DEL,DEL2,DEL3,DEL31,DEL4,DEL41,DEL5,
     *W,W2,W21,W3,W31,W4,W41,W5,EFD,EFD2,EFD3,EFD4,EFD5,
     *DIQ2,DIQ31,DIQ3,DIQ4,DIQ5,DID2,DID3,DID31,DID4,DID41,DID5
     *,DDEL2,DDEL3,DDEL31,DDEL4,DDEL41,DDEL5,DW,DW2,
     *DW21,DW3,DW31,DW4,DW41,DW5,DFD2,DFD3,DFD4,DFD5)
      REAL IQ(803),ID(803),DEL(803),W(803),EFD(803),TIME(803)
      REAL DEL2(803),W2(803),EFD2(803),ID2(803),W21(803),IQ2(803)
      REAL DEL3(803),IQ3(803),ID3(803),W3(803),DEL31(803),EFD3(803)
      REAL ID31(803),W31(803),IQ31(803)
      REAL ID4(803),EFD4(803),DEL4(803),W4(803),DEL41(803),W41(803)
      REAL ID41(803),IQ4(803)
      REAL IQ5(803),ID5(803),DEL5(803),W5(803),EFD5(803)
      REAL DDEL2(803),DW2(803),DFD2(803),DID2(803),DW21(803)
      REAL DDEL3(803),DIQ3(803),DID3(803),DW3(803),DDEL31(803)
      REAL DID31(803),DW31(803),DIQ31(803),DIQ2(803),DFD3(803)
      REAL DID4(803),DFD4(803),DDEL4(803),DW4(803),DDEL41(803)
      REAL DID41(803),DIQ4(803),DW41(803)
      REAL DIQ5(803),DID5(803),DDEL5(803),DW5(803),DFD5(803)
      EIQ=0.
      EIQ2=0.
      EIQ3=0.
      EIQ31=0.
      EIQ4=0.
      EIQ5=0.
      EID=0.
      EID2=0.
      EID3=0.
```

```
EID31=0.
EID4=0.
EID41=0.
EID5=0.
EDEL=0.
EDEL2=0.
EDEL3=0.
EDEL31=0.
EDEL4=0.
EDEL41=0.
EDEL5=0.
EW=0.
EW2=0.
EW21=0.
EW3=0.
EW31=0.
EW4=0.
EW41=0.
EW5=0.
VFD=0.
VFD2=0.
VFD3=0.
VFD4=0.
VFD5=0.
DO 33 I=1,801
EIQ2=EIQ2+(IQ(I)-IQ2(I))**2
EIQ3=EIQ3+(IQ(I)-IQ3(I))**2
EIQ31=EIQ31+(IQ(I)-IQ31(I))**2
EIQ4=EIQ4+(IQ(I)-IQ4(I))**2
EIQ5=EIQ5+(IQ(I)-IQ5(I))**2
EID2=EID2+(ID(I)-ID2(I))**2
EID3=EID3+(ID(I)-ID3(I))**2
EID31=EID31+(ID(I)-ID31(I))**2
EID4=EID4+(ID(I)-ID4(I))**2
EID41=EID41+(ID(I)-ID41(I))**2
EID5=EID5+(ID(I)-ID5(I))**2
EDEL2=EDEL2+(DEL(I)-DEL2(I))**2
EDEL3=EDEL3+(DEL(I)-DEL3(I))**2
EDEL31=EDEL31+(DEL(I)-DEL31(I))**2
EDEL4=EDEL4+(DEL(I)-DEL4(I))**2
EDEL41=EDEL41+(DEL(I)-DEL41(I))**2
EDEL5=EDEL5+(DEL(I)-DEL5(I))**2
EW2=EW2+(W(I)-W2(I))**2
EW21=EW21+(W(I)-W21(I))**2
EW3=EW3+(W(I)-W3(I))**2
EW31=EW31+(W(I)-W31(I))**2
EW4=EW4+(W(I)-W4(I))**2
EW41=EW41+(W(I)-W41(I))**2
```

```
      EW5=EW5+(W(I)-W5(I))**2
      VFD2=VFD2+(EFD(I)-EFD2(I))**2
      VFD3=VFD3+(EFD(I)-EFD3(I))**2
      VFD4=VFD4+(EFD(I)-EFD4(I))**2
33    VFD5=VFD5+(EFD(I)-EFD5(I))**2
      EIQ2=SQRT(EIQ2/800)
      EIQ3=SQRT(EIQ3/800)
      EIQ31=SQRT(EIQ31/800)
      EIQ4=SQRT(EIQ4/800)
      EIQ5=SQRT(EIQ5/800)
      EID2=SQRT(EID2/800)
      EID3=SQRT(EID3/800)
      EID31=SQRT(EID31/800)
      EID4=SQRT(EID4/800)
      EID41=SQRT(EID41/800)
      EID5=SQRT(EID5/800)
      EDEL2=SQRT(EDEL2/800)
      EDE2C=SQRT(EDE2C/800)
      EDEL3=SQRT(EDEL3/800)
      EDEL31=SQRT(EDEL31/800)
      EDEL4=SQRT(EDEL4/800)
      EDEL41=SQRT(EDEL41/800)
      EDEL5=SQRT(EDEL5/800)
      EW2=SQRT(EW2/800)
      EW21=SQRT(EW21/800)
      EW3=SQRT(EW3/800)
      EW31=SQRT(EW31/800)
      EW4=SQRT(EW4/800)
      EW41=SQRT(EW41/800)
      EW5=SQRT(EW5/800)
      VFD2=SQRT(VFD2/800)
      VFD3=SQRT(VFD3/800)
      VFD4=SQRT(VFD4/800)
      VFD5=SQRT(VFD5/800)
      MB=2
      MC=3
      MD=4
      ME=5
      MF=6
      MG=7
      WRITE(6,140)
      WRITE(6,139)
      WRITE(6,138)
      WRITE(6,139)
      WRITE(6,137)MB,EIQ2,EW21,MF,MG
      WRITE(6,136)MB,EDEL2,EW2,MF,MG
      WRITE(6,135)MB,EID2,VFD2,MF,MG
      WRITE(6,134)MC,EIQ3,EDEL3,EW3,MF,MG,ME
```

```
      WRITE(6,133)MC,EIQ31,EID3,EDEL31,MF,MG,ME
      WRITE(6,132)MC,EID31,VFD3,EW31,MB,MC,ME
      WRITE(6,130)MD,EIQ4,EID41,EDEL41,EW41,MF,MG,MB,MC
      WRITE(6,131)MD,EID4,VFD4,EDEL4,EW4 ,MF,MG,MB,MC
      WRITE(6,129)ME,EIQ5,EID5,VFD5,EDEL5,EW5,MF,MG,MB,MC,ME
      WRITE(6,139)
  140 FORMAT('1',5X,'R.M.S ERROR OF REDUCED , CLASSICAL AND
     *COMPANSATED AND CLASSICAL MODEL OF A GENERATING UNIT',/)
  138 FORMAT(3X,'ORDER OF MODEL',15X,'IQ',12X,'ID',14X,'FIELD-
     *VOL.',8X,'DELTA',9X,'FREQ.',9X,'DOMINANT EIGENVALUES',/)
  137 FORMAT(8X,I2,17X,F10.6,10X,'---',15X,'---',14X,'---',
     *8X,F10.6,7X,I2,',',I2,/)
  136 FORMAT(8X,I2,22X,'---',12X,'---',15X,'---',9X,F10.6,6X,
     *F10.6,7X,I2,',',I2,/)
  135 FORMAT(8X,I2,22X,'---',8X,F10.6,7X,F10.6,12X,'---',12X,
     *'---',10X,I2,',',I2,/)
  134 FORMAT(8X,I2,17X,F10.6,10X,'---',15X,'---',9X,F10.6,6X,
     *F10.6,7X,I2,',',I2,',',I2,/)
  133 FORMAT(8X,I2,17X,F10.6,6X,F10.6,12X,'---',9X,F10.6,10X,
     *'---',10X,I2,',',I2,',',I2,/)
  132 FORMAT(8X,I2,22X,'---',8X,F10.6,7X,F10.6,11X,'---',9X,
     *F10.6,7X,I2',',I2,',',I2,/)
  131 FORMAT(8X,I2,22X,'---',8X,F10.6,7X,F10.6,7X,F10.6,6X,
     *F10X,7X,I2,',',I2,',',I2,',',I2,/)
  130 FORMAT(8X,I2,17X,F10.6,6X,F10.6,12X,'---',9X,F10.6,
     *6X,F10.6,7X,I2,',',I2,',',I2,',',I2,/)
  129 FORMAT(8X,I2,17X,F10.6,6X,F10.6,7X,F10.6,7X,F10.6,
     *6X,F10.6,7X,I2,',',I2,',',I2,',',I2,',',I2,/)
  139 FORMAT('0',130('-'))
C CALCULATION OF THE ERROR VECTOR FOR EACH OUTPUT.
      DO 333 I=1,801
      DIQ2(I)=(IQ(I)-IQ2(I))
      DIQ3(I)=(IQ(I)-IQ3(I))
      DIQ31(I)=(IQ(I)-IQ31(I))
      DIQ4(I)=(IQ(I)-IQ4(I))
      DIQ5(I)=(IQ(I)-IQ5(I))
      DID2(I)=(ID(I)-ID2(I))
      DID3(I)=(ID(I)-ID3(I))
      DID31(I)=(ID(I)-ID31(I))
      DID4(I)=(ID(I)-ID4(I))
      DID41(I)=(ID(I)-ID41(I))
      DID5(I)=(ID(I)-ID5(I))
      DDEL2(I)=(DEL(I)-DEL2(I))
      DDEL3(I)=(DEL(I)-DEL3(I))
      DDEL31(I)=(DEL(I)-DEL31(I))
      DDEL4(I)=(DEL(I)-DEL4(I))
      DDEL41(I)=(DEL(I)-DEL41(I))
      DDEL5(I)=(DEL(I)-DEL5(I))
```

```
      DW2(I)=(W(I)-W2(I))
      DW21(I)=(W(I)-W21(I))
      DW3(I)=(W(I)-W3(I))
      DW31(I)=(W(I)-W31(I))
      DW4(I)=(W(I)-W4(I))
      DW41(I)=(W(I)-W41(I))
      DW5(I)=(W(I)-W5(I))
      DFD2(I)=(EFD(I)-EFD2(I))
      DFD3(I)=(EFD(I)-EFD3(I))
      DFD4(I)=(EFD(I)-EFD4(I))
 333  DFD5(I)=(EFD(I)-EFD5(I))
      RETURN
      END
/*
//GO.SYSIN DD *
13
1.563 1.653 1.470 1.560 1.5020 1.6080 1.646 0.20
0.0032 0.001 0.011 0.014 0.02 0.3220 0.9460 1.560
1.00 1.030 0.51 1.130 0.5986 377.0 0.014 18.850 0.2500 1.00 0.0
13.89 50.0 1.0 0.057 2.028 0.02 0.001 0.450
/*
//
```